MAC 1105 Lecture Outline

Some of this beginning material will be familiar to you, but these are essential algebra skills needed throughout our course. **Please be careful about judging the difficulty of the entire course based on the first few days of material.** If your ALEKS score qualifies you for MAC 1114, 1140, or 2233, then take a look at the complete outline before deciding to skip MAC 1105.

Objective 1: Multiplying Polynomials

Multiply two polynomials by repeatedly using the Distributive Property and the Laws of Exponents. That is, multiply each term of one polynomial by each term of the other. We call it **FOIL**. (First, Outer, Inner, Last.) Combine like terms.

Be sure to note how the final product is constructed from the two binomial factors. In the next section, we will use these concepts to factor.

Use these formulas for Special Products:

Difference of Squares: $(x-a)(x+a) =$ ____________________

Squares of Binomials, or Perfect Squares:

$(x+a)^2 =$ ____________________ $(x-a)^2 =$ ____________________

Multiply. Simplify your answer.

$(2x+4)(3x-1)$

$(-3x+1)(2-x)$

$(2x+5z)(6x-7z)$

$(u+9)(u-9)$

$(3x+1)(3x-1)$

Rewrite without parentheses and simplify.

$(y+2)^2$

$(2v+7w)^2$

Objective 2a: Factoring Polynomials

Factoring out the GCF.

First, you always want to see if there is a common factor. If so, factor it out first.

Factor the following expressions.

$4x^2 - 24x$ $3y^2 + 3$ $16uv^3y^7 + 28u^6v^9$

Write the expression by factoring out the common binomial.

$3y^2(y + 8) + 2(y + 8)$ $7x^2(x - 3) + (x - 3)$

Factoring by Grouping:

This technique is most typically applied in a polynomial of 4 terms.

1. Group the first two terms together and factor out the GCF.

2. Group the last two terms together and factor out the GCF. (Rearrange the terms, when needed.)

The resulting binomials must match.

3. Factor out the GCF (which will be the common binomial).

4. Write what's left, inside parentheses.

Factor by Grouping.

$5v^3 + 2v^2 + 35v + 14$

$4u^3 + 5u^2 - 28u - 35$

$xw - 3x - 2w + 6$

$vy - 14v - 2v^2 + 7y$

Factoring Difference of Squares.

$w^2 - 49$

$9z^2 - 64$

$4 - 25v^2$

Objective 2b: More Factoring Polynomials

Factoring Trinomials:

There are two common techniques for factoring a second degree polynomial of the form $Ax^2 + Bx + C$. **It is critical that you practice and determine the technique that is best for you.**

The **FOIL-method** is usually the most efficient method when $A = 1$. For $x^2 + Bx + C$, find integers whose product is C and whose sum is B. The FOIL-method can certainly be applied when $A \neq 1$ and A, B, C have no common factors. In this case, find 4 integers a, b, c, and d so that $ac = A$, $bd = C$, and $ad + bc = B$. $Ax^2 + Bx + C = (ax + b)(cx + d)$

The **AC-method** for factoring $Ax^2 + Bx + C$, $A \neq 1$ and A, B, C have no common factors:

Step 1: Find the value of AC.

Step 2: Find the factors of AC, a and b, whose sum is B.

Step 3: Write $Ax^2 + Bx + C = Ax^2 + ax + bx + C$.

Step 4: Factor this expression by the Factor by Grouping technique.

Factor.

$y^2 - 7y + 12$

$x^2 - 11x + 18$

Factor.

$2z^2 - 5z + 3$

$9z^2 - 4z - 7$

Factor.

$2x^2 + 28x + 48$

$6v^2 - 66v - 72$

Factoring expressions that occur in calculus:

In calculus, once the derivative is computed by the product rule and chain rule, expressions like these occur and are simplified by using the factoring techniques we have learned. Simply first, when needed, then factor out the greatest common factor.

Factor completely.

$7(3x+2)^2(x-1)^2+(3x+2)(x-1)^3$

$2(8x-7)^2(4x-9)+(8x-7)(4x-9)^2$

Factor completely.

$(9x + 10)^2 - 4x(9x + 10)$

$3(2x - 3)^2(5x - 6) + (2x - 3)(5x - 6)^2$

Objective 3a: Simplifying Rational Expressions

A **Rational Expression** is a quotient of polynomials.

Relationship between $a-b$ and $b-a$. These expressions are additive inverses.

$b - a =$ ______________________ $a - b =$ ______________________

Simplify, using this relationship. $\dfrac{b-a}{a-b} =$

Also recall (for $a \neq 0$): $\dfrac{0}{a} =$ _____ and $\dfrac{a}{0} =$ ___________

For each expression, simplify if possible.

$\dfrac{8u-3}{3-8u}$ $\dfrac{4u+3}{3u+4}$ $\dfrac{v-5}{-5+v}$

Reducing a rational expression to lowest terms:

Factor the numerator and denominator and cancel any common factors using the Cancellation Property: $\dfrac{ac}{bc} =$

Simplify.

$$\frac{6x^2 - 9x}{3x^2 - 15x}$$

$$\frac{w - 5}{w^2 - 10w + 25}$$

$$\frac{w^2 + 9w + 14}{2w^2 - 98}$$

$$\frac{w^2 - 5w + 4}{5 - 5w^2}$$

$$\frac{4y^2 - 9}{2y^2 - y - 3}$$

$$\frac{45(3y-4)^3(y-6)}{18(y-6)^5(3y-4)^2}$$

Objective 3b: Multiplying and Dividing Rational Expressions

Factor completely before performing any other operations. Simplify.

$$\frac{a}{b} \cdot \frac{c}{d} =$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} =$$

Multiply or Divide. Simplify your answer as much as possible.

$$\frac{2x-4}{3} \cdot \frac{7}{8x-16}$$

$$\frac{x^2+5x+6}{4x-12} \cdot \frac{x-3}{x^2+2x-3}$$

Multiply or Divide. Simplify your answer as much as possible.

$$\frac{x^2+x-6}{x^2+4x-5}\cdot\frac{x^2-25}{x^2+2x-15}$$

$$\frac{9x^2-3x-2}{2x-6}\cdot\frac{x-3}{9x^2-4}$$

Multiply or Divide. Simplify your answer as much as possible.

$$\frac{4x^2 - 12x - 7}{5x + 5} \cdot \frac{x + 1}{4x^2 - 49}$$

$$\frac{\dfrac{4x - 12}{5}}{\dfrac{8x - 24}{3}}$$

Multiply or Divide. Simplify your answer as much as possible.

$$\frac{\frac{x-2}{x^2+4x+3}}{\frac{4x-8}{x^2+2x-3}}$$

Multiply or Divide. Simplify your answer as much as possible.

$$\frac{\frac{21x^2 - 2x - 3}{4x - 12}}{\frac{49x^2 - 9}{x - 3}}$$

Objective 3c: Adding and Subtracting Rational Expressions

Fractions must have a common denominator in order to be added or subtracted.

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$$

When needed, use the LCM Method for obtaining a common denominator: Factor completely each denominator. The LCM of the denominators is the product of each of these factors raised to a power equal to the greatest number of times that the factor occurs in the polynomials. Write each rational expression using the LCM as the common denominator. Add or Subtract as indicated above.

Add or Subtract. Simplify your answer as much as possible.

$$\frac{7z - 4}{8z} + \frac{3z + 2}{8z}$$

Add or Subtract. Simplify your answer as much as possible.

$$\frac{x+9}{x-1} - \frac{x-3}{x}$$

$$\frac{6}{4x-5} - \frac{7}{5-4x}$$

Add or Subtract. Simplify your answer as much as possible.

$$\frac{2}{x-6}+\frac{3}{x+5}$$

$$\frac{x}{7x-6}-\frac{x+2}{8x}$$

Add or Subtract. Simplify your answer as much as possible.

$$-\frac{4c-8d}{5c}-\frac{5c+3d}{5c}$$

$$\frac{13}{4x+8}-\frac{2}{x+2}$$

Add or Subtract. Simplify your answer as much as possible.

$$\frac{4}{3x^2+16x+21}+\frac{2}{3x^2+13x+14}$$

Objective 3d: Simplifying Complex Rational Expressions

When sums or differences of rational expressions appear as the numerator or denominator of a quotient, the quotient is called a complex rational expression. (That is, a fraction made up of fractions.)

Treat the numerator and denominator of the complex rational expression separately, performing whatever operations are indicated and simplify the result. Then simplify the resulting rational expression. (That is, invert and multiply.)

Simplify.

$$\frac{5 - \dfrac{3}{2w}}{\dfrac{3}{2w} - 7}$$

Simplify.

$$\frac{1+\frac{5}{x}}{1-\frac{25}{x^2}}$$

Simplify.

$$\frac{9-\frac{9}{v}}{9-\frac{9}{v-1}}$$

Simplify.

$$\frac{1 - \dfrac{w+3}{4w+24}}{2 + \dfrac{2}{w+6}}$$

Objective 4a: Simplifying Radicals

To **simplify** radicals, find the prime factorization of the number inside the radical. Then, determine the index of the radical. The index tells you how many of a kind you need to put together to be able to move that number or variable from inside the radical to outside the radical.

For example, if the index is 2 (a square root), then you need two of a kind to move from inside the radical to outside the radical. If the index is 3 (a cube root), then you need three of a kind to move from inside the radical to outside the radical.

Move each group of numbers or variables from inside the radical to outside the radical. If there are not enough numbers or variables to make a group then leave those numbers or variables inside the radical. *Note: each group of numbers or variables gets written **once** when they move outside the radical because they are now one group.

Simplify the expressions both inside and outside the radical by multiplying. Multiply all the numbers and variables inside the radical together. Multiply all the numbers and variables outside the radical together.

Simplify. Assume the variable represents a positive real number.

$\sqrt{54}$

$\sqrt{180}$

$\sqrt{y^{14}}$

$\sqrt{64y^{12}}$

$\sqrt[3]{64u^{12}}$

$\sqrt{w^{15}}$

Simplify. Assume the variable represents a positive real number.

$\sqrt{24y^{13}}$

$\sqrt{28y^{12}z^7}$

$\sqrt[4]{162y^9}$

$\sqrt[5]{192y^{12}z^5}$

Objective 4b: Rational Exponents

Expressions with rational exponents may be written in terms of radicals using the following definitions.

If a is a real number and $n \geq 2$ is an integer, then $a^{1/n} =$ __________ Provided the root exists (i.e. is a real number).

If a is a real number and m and n are integers containing no common factors, with $n \geq 2$, then $a^{m/n} =$ ____________________

That is, the numerator of the rational exponent is the power on the radicand;

the denominator of the rational exponent is the index on the radical.

The symbol $\sqrt[n]{a}$ for the principal nth root of a is called a ______________; the integer n is called the ______________, and a is called the ______________.

If n is an even integer, for $n \geq 2$

then, $\sqrt[n]{positive\ \ number} =$ ______________

then, $\sqrt[n]{negative\ \ number} =$ ______________

If n is an odd integer, for $n \geq 3$

then, $\sqrt[n]{positive \ \ number} =$ ____________

then, $\sqrt[n]{negative \ \ number} =$ ____________

Note:

1. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$ are equivalent statements.

2. $a^{-n} =$ _________

3. $a^{-n/m} =$ _________

4. $ax^2 \neq (ax)^2$

Write the following as an exponential expression.

$\sqrt[7]{x^5}$ $\sqrt[8]{y^3}$ $\sqrt[12]{y^7}$

Evaluate.

$(8)^{1/3}$ $\qquad$ $16^{1/4}$

Simplify. Write your answers without exponents.

$(-64)^{1/3}$ $\qquad$ $(-49)^{1/2}$ $\qquad$ $16^{3/4}$

Simplify. Write your answers without exponents.

$16^{-3/2}$

$-25^{-1/2}$

$(-81)^{-3/4}$

$-125^{-1/3}$

$(-125)^{-2/3}$

Simplify. Write your answers without exponents.

$$\left(-\frac{8}{27}\right)^{-2/3}$$

$$-\left(\frac{64}{27}\right)^{-2/3}$$

Objective 4c: Radical Operations

To **add or subtract** radicals, the indices (the index of the radical) and what is inside the radical (radicand) must be __________________.

If the indices and radicands are the same, then add or subtract the terms in front of each like radical. If the indices or the radicands are not the same, then you cannot add or subtract the radicals.

$2\sqrt[3]{x} + 5\sqrt[3]{x} =$ ____________________

To **multiply** radicals, the indices (the index of the radical) must be __________________.

If the indices are the same, then multiply the numbers outside the radical together and multiply the numbers inside the radical together. Remember to simplify the radical if possible.

$(4\sqrt{2})(2\sqrt{3}) =$ ____________________

Simplify.

$8\sqrt{7} - 6\sqrt{7}$

$4\sqrt{18} + \sqrt{50}$

$2\sqrt{5} + 4\sqrt{45} - \sqrt{20}$

Simplify.

$\sqrt{3} \cdot \sqrt{5}$

$\sqrt{15} \cdot \sqrt{3}$

$4\sqrt{72} \cdot \sqrt{24}$

$\sqrt{3x} \cdot \sqrt{2x}$

Simplify.

$\sqrt{10z^4} \cdot \sqrt{5z^5}$

$\sqrt{2u^2x} \cdot \sqrt{8u^3x}$

$\sqrt[4]{12} \cdot \sqrt{4}8$

$\sqrt[3]{8v^3} \cdot \sqrt[3]{16v}$

Multiply. Simplify your answer as much as possible

$\sqrt{7}(\sqrt{3}+6)$

$\sqrt{2}(\sqrt{3}-3\sqrt{10})$

$(2\sqrt{10}-1)(4-2\sqrt{2})$

Objective 4d: Rationalize the Denominator

Rationalizing the Denominator. In most circumstances, when radicals occur in the denominator of a rational expression, the quotient is rewritten so that the new denominator contains no radicals.

Multiply the numerator and denominator by an appropriate expression so that no radical remains in the denominator.

When the denominator contains a binomial expression, then a special construction is needed to eliminate the radical. This special construction is based on the Difference of Squares formula: $(a-b)(a+b) =$ ______________

Rationalize the denominator and simplify.

$\dfrac{\sqrt{2}}{\sqrt{3}}$ $\qquad$ $\sqrt{\dfrac{6}{5}}$ $\qquad$ $\dfrac{\sqrt[3]{2}}{\sqrt[3]{9}}$

Rationalize the denominator and simplify.

$$\frac{\sqrt{2}}{\sqrt{3x}}$$

$$\frac{7}{3\sqrt{2}-1}$$

Rationalize the denominator and simplify.

$$\frac{\sqrt{11}-\sqrt{5}}{\sqrt{11}+\sqrt{5}}$$

$$\frac{-3}{7+3\sqrt{y}}$$

Objective 5a: Linear Equations

A **linear equation** with one variable is an equation equivalent in form to $ax+b=0$, a and b are integers and $a \neq 0$.

Solving Linear Equations:

Use the distributive property to remove grouping symbols; Combine like terms; Isolate variable.

For rational equations, multiply both sides of the equation by the LCM of all the denominators. (This technique may also be used when there are fractional coefficients or constants.)

When solving rational equations, we must note any values that create division by zero. These are restrictions on the variable and cannot be included in the solution set.

Solve each equation. Simplify your answer as much as possible.

$$-7 = \frac{5x - 17}{-4}$$

$$-8y + 5(y - 5) = 4$$

Solve each equation. Simplify your answer as much as possible.

$2y + 12 = 6(y + 6)$

$7(x - 3) - 2 = -5(-4x + 5) - 3x$

Solve each equation. Simplify your answer as much as possible.

$$\frac{x+5}{6} = \frac{5}{4}$$

$$\frac{-14}{x-4} = \frac{-6}{x}$$

Objective 5b: Linear Equation Applications

A garden table and a bench cost \$600 combined. The cost of the garden table is three times the cost of the bench. What is the cost of the bench?

Pamel's age is two times Jiri's age. The sum of their ages is 51. What is Jiri's age?

Two trains leave the station at the same time, one heading west and the other east. The westbound train travels at 70 mph. The eastbound train travels at 90 mph. How long will it take for the two trains to be 416 miles apart? Do not do any rounding.

Two trains leave stations 476 miles apart at the same time and travel toward each other. One train travels at 95 miles per hour while the other travels at 75 miles per hour. How long will it take for the two trains to meet?

Aldo drove to the mountains last weekend. There was heavy traffic on the way there, and the trip took 10 hours. When Aldo drove home, there was no traffic and the trip only took 7 hours. If his average rate was 18 miles per hour faster on the trip home, how far away does Aldo live from the mountains?

Objective 5c: Linear Inequalities

Interval notation is used to express the solutions to inequalities.

Interval notation is always written in ______________________________.

Inequalities are easier to read when the variable is on the _____________

Note: $\leq, \geq$ are _____________, use _____________

$<, >$ are _____________, use _____________

$\pm\infty$ are _____________, use _____________

Inequality	Graph	Interval Notation	Set Notation
$x \geq a$			
$x > a$			
$x \leq a$			
$x < a$			
$a < x < b$			
$a \leq x \leq b$			
$a \leq x < b$			
$a < x \leq b$			
All real numbers			

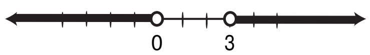
Graph and write each inequality using interval notation.

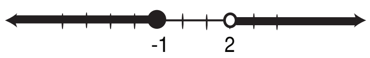
$-2 < x \leq 0$ $\quad$ $1 < x$ $\quad$ $x \geq -5$

Graph and write each interval as an inequality.

$[0, 1)$ $\quad$ $(-\infty, 5]$ $\quad$ $(-3, \infty)$

Write a compound inequality given the graph on a number line. Use x as the variable.



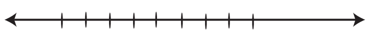


Graph the compound inequality on the number line

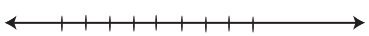
$x > 5$ or $x \leq 3$

$x \leq -2$ or $x > 4$

$x < 7$ and $x \geq 4$



$x > 0$ and $x \leq 7$



Solving Inequalities:

Like linear equations, use the distributive property to remove grouping symbols, combine like terms, isolate the variable. **Unlike** linear equations, when you multiply/divide both sides of an inequality by a negative number, you must __________ the inequality sign.

Solving Combined Inequalities. Inequalities of the form $p < ax + b < q$ may be solved by separating into two separate inequalities and then finding the intersection of the solution sets. That is, solve $p < ax + b$ and $ax + b < q$.

Inequalities of the form $p < ax + b < q$ may also be solved by isolating the variable in the middle, solving the two inequalities at the same time.

Recall that fractional coefficients may be removed by multiplying every term on each side by the LCM of the denominators.

Solve each inequality. Simplify your answer as much as possible.

$2x + 2 \leq -4$

$-\frac{4}{5}u \geq -10$

Solve each inequality. Simplify your answer as much as possible.

$-38 < 5x - 8$

$-12 - 4w > 12$

Solve each inequality. Simplify your answer as much as possible.

$10 > -\frac{2}{3}v + 2$

$5 - 4x > 13 - 2x$

Solve each inequality. Simplify your answer as much as possible.

$5x - 24 < -2(3 - 4x)$

Objective 6: Quadratic Equations

A **quadratic equation** is an equation equivalent to the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$. When written in this form the equation is said to be in **Standard Form**.

We'll look at three ways to solve quadratics: factoring, square root method, and quadratic formula.

Solving a Quadratic Equation by Factoring:

Once factored as the product of two binomial factors, use the **Zero Factor Property** to obtain solution(s). If $ab = 0$, then ______________.

Solve for the variable. If there is more than one solution, separate them with commas.

$(9 - u)(2u - 3) = 0$

$4x^2 - 12x = 0$

Solve for the variable. If there is more than one solution, separate them with commas.

$u^2 + 5u - 14 = 0$

$5u^2 + 5 = -26u$

Solving a Quadratic Equation by the Square Root Method:

If $x^2 = p$ and $p \geq 0$, then $x = \pm\sqrt{p}$.

If $(x \pm a)^2 = p$, then $x \pm a = \pm\sqrt{p}$.

Solve. Simplify your answer as much as possible.

$x^2 = 36$

$x^2 = 28$

$(x + 2)^2 = 1$

$(3z - 2)^2 = 4$

Solve. Simplify your answer as much as possible.

$(4z - 3)^2 = 5$

$(y + 3)^2 - 63 = 0$

Solving a Quadratic Equation by the Quadratic Formula:

For $ax^2 + bx + c = 0$, $a \neq 0$: $x =$

The quantity $b^2 - 4ac$ is called the ______________________.

If $b^2 - 4ac > 0$, there are ________________________

If $b^2 - 4ac = 0$, there is ________________________

*In this case, the quadratic equation would factor as a perfect square. This is called "one repeated solution".

If $b^2 - 4ac < 0$, there are ________________________

Use the quadratic formula to solve for the variable.

$2x^2 + 7x - 6 = 0$

$6x^2 + 4x = 1$

Use the quadratic formula to solve for the variable.

$2x^2 = 1 - 2x$

Compute the value of the discriminant and give the number of real solutions of the quadratic equation.

$3x^2 - x + 2 = 0$ $\quad$ $25x^2 - 20x + 4 = 0$ $\quad$ $2x^2 - 3x - 7 = 0$

Objective 7: Radical Equations

Solving a Radical Equation. Isolate the radical, then raise both sides of the equation to a power equal to the index.

When the index is even, we must remember: $\sqrt[n]{a^n} = |a|$. That is, an even-root radical always answer the principal (positive) root. When raising both sides of the equation to an **even** power, **Extraneous solutions** are possible. Extraneous solutions do not meet the above condition.

Any time you raise both sides of an equation to an ________________ power, the solutions must be ________________ to see if they meet the condition mentioned above.

Solve for the variable, where the variable is a real number.

$\sqrt[3]{5x-1} - 4 = 0$

$\sqrt[5]{x^2+2x} = -1$

Solve for the variable, where the variable is a real number.

$\sqrt{x+5} = 8$

$1 = \sqrt{3y+13} - 1$

Solve for the variable, where the variable is a real number.

$\sqrt{3w+14}=\sqrt{5w+2}$

$\sqrt{6u-8}=u$

Solve for the variable, where the variable is a real number.

$\sqrt{20 - 2v} = v + 2$

Solve for the variable, where the variable is a real number.

$(3x - 5)^{1/2} = 2$

$(3u + 2)^{1/3} + 4 = 6$

Objective 8: Coordinate Plane

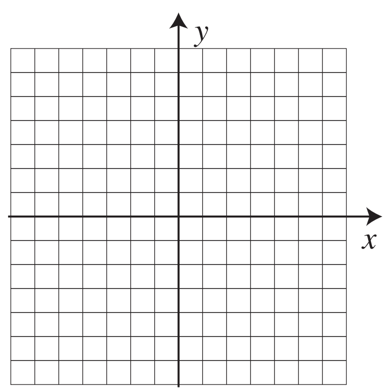
Rectangular Coordinates The Rectangular (or Cartesian) Coordinate System consists of a horizontal number line and a vertical number line.

The lines intersect at 0 on each and this point is called the _________.

The horizontal number line is called the _________ and the positive direction is to the _________, as indicated by the arrow on the end of the number line.

The vertical number line is called the _________ and the positive direction is _________, as indicated by the arrow on the end of the number line.

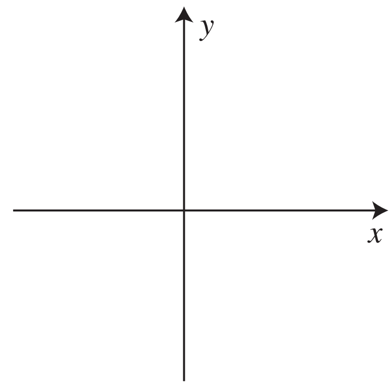
The coordinate axes divide the coordinate plane into 4 regions called ______________ and they are numbered with Roman Numerals, counterclockwise, as given below.



The notation $P = (x, y)$ is used to refer to a point in the coordinate plane. It is an

____.

Distance Formula

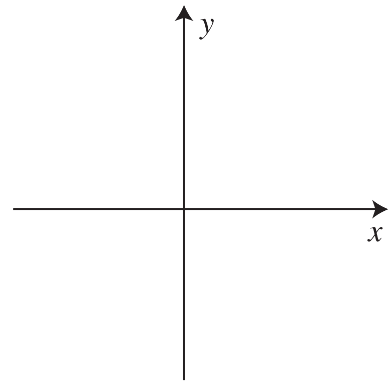


Find the distance between points P_1 and P_2

$P_1 = (2, -2)$; $P_2 = (9, -7)$

$P_1 = (-4, -3)$; $P_2 = (6, 2)$

Mid-Point Formula



Find the midpoint of the line segment joining points P_1 and P_2

$P_1 = (5, -4)$; $P_2 = (-3, 8)$

$P_1 = (-4, -3)$; $P_2 = (2, 2)$

Determining Whether a Point is on the Graph of an Equation. If the point (a, b) is on the graph of an equation, then when __________ and __________, the equation is satisfied.

Examples:

Fill in the values using the function rule given.

$y = -6x + 1$

$x = -1$ $\quad$ $y =$ __________

$x = 0$ $\quad$ $y =$ __________

$x = 1$ $\quad$ $y =$ __________

$x = 5$ $\quad$ $y =$ __________

Determine whether the given points are on the graph of the equation.

Equation: $4x + 5y = -13$

Points: $(8, -9)$; $(-7, 4)$; $(6, 3)$; $(-2, -1)$

Objective 9a: Intercepts and Symmetry

Finding Intercepts. The points, if any, where a graph crosses or touches the coordinate axes are called ____________.

The x-coordinate of a point at which the graph crosses or touches the x-axis is an ____________.

The y-coordinate of a point at which the graph crosses or touches the y-axis is a ____________.

To find x-intercepts from an equation, let ______________________________.

To find y-intercepts from an equation, let ______________________________.

Find the x-intercept and y-intercept of the line.

$6x + 4y = -24$

$8x - 5y = 14$

Find the x-intercept and y-intercept of the line.

$9x^2 + 36y^2 = 49$

$y^2 + x - 36 = 0$

Find the x-intercept and y-intercept of the line.

$$y = \frac{25 - x^2}{7 + x^2}$$

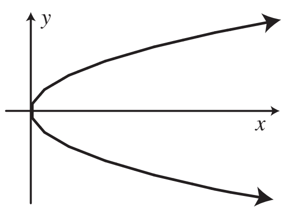
$$y = \frac{x^2 - 49}{x^2 + 5}$$

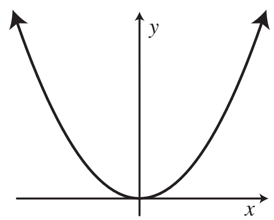
Symmetry

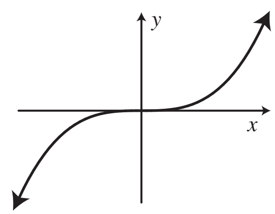
A graph is said to be **Symmetric with respect to the** x**-axis** if, for every point (x, y) on the graph, the point ________ is also on the graph.

A graph is said to be **Symmetric with respect to the** y**-axis** if, for every point (x, y) on the graph, the point ________ is also on the graph.

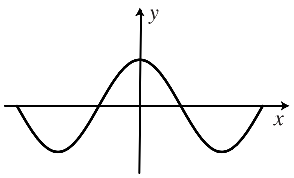
A graph is said to be **Symmetric with respect to the origin** if, for every point (x, y) on the graph, the point ________ is also on the graph.

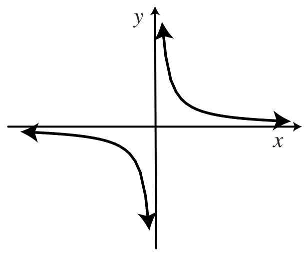


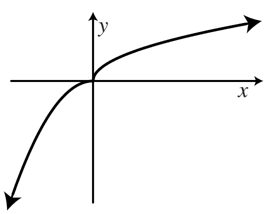


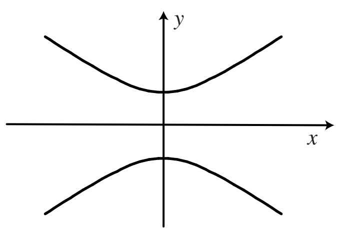


For each graph, Indicate whether the graph is symmetric with respect to the x-axis, the y-axis, the origin, or has no symmetry.









Tests for Symmetry

To test the graph of an equation for Symmetry with respect to the

x-Axis **Replace** ______________ in the equation, **simplify**, see if an equivalent equation results.

y-Axis **Replace** ______________ in the equation, **simplify**, see if an equivalent equation results.

Origin **Replace** ______________ in the equation, **simplify**, see if an equivalent equation results.

For each equation, determine whether its graph is symmetric with respect to the x-axis, the y-axis, and the origin.

$y^2 = x + 9$

For each equation, determine whether its graph is symmetric with respect to the x-axis, the y-axis, and the origin.

$27x^2 + 15y^2 = 48$

For each equation, determine whether its graph is symmetric with respect to the x-axis, the y-axis, and the origin.

$x^2 - y + 2 = 0$

For each equation, determine whether its graph is symmetric with respect to the x-axis, the y-axis, and the origin.

$x = y^2 - 10$

For each equation, determine whether its graph is symmetric with respect to the x-axis, the y-axis, and the origin.

$x^2y^2 + xy = 4$

Objective 9b: Slope

Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, then the **slope m** of the nonvertical line L containing P and Q is defined:

If $x_1 = x_2$, meaning the denominator of the slope calculation is equal to 0, then L is a ________________ and the slope is ___________________.

If $y_1 = y_2$, meaning the numerator of the slope calculation is equal to 0, then L is a ________________ and the slope is ____________.

When the slope of a line is ________________, the line slants upward from left to right.

When the slope of a line is ________________, the line slants downward from left to right.

Find the slope of the line containing the given points.

$(-2,1);(-3,-4)$ $\quad$ $(7,0);(4,2)$ $\quad$ $(4,2);(-5,2)$ $\quad$ $(2,0);(2,2)$

Objective 9c: Writing Equations of Lines

Point-Slope Form of an Equation of a Line An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

Slope-Intercept Form of an Equation of a Line An equation of a line with slope m and y-intercept b is

General Form of an Equation of a Line The equation of a line is in General Form (or Standard Form) when it is written as

Equation of a Horizontal Line A horizontal line is given by an equation of the form __________ where a is the y-intercept.

Equation of a Vertical Line A vertical line is given by an equation of the form __________ where b is the x-intercept.

Find the slope and the y-intercept of the line.

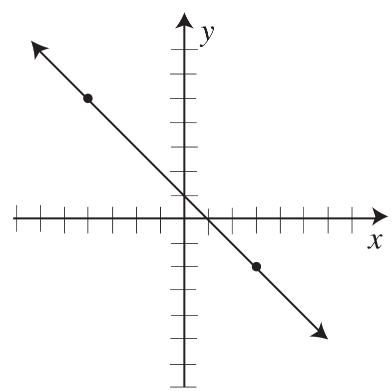
$y = 3x - 8$ $\quad$ $-9x + 3y = -3$ $\quad$ $-3x - 4y = 12$

Write an equation in slope-intercept form for the line with slope $\frac{3}{5}$ and y-intercept -3.

A line passes through the point $(8, 2)$ and has a slope of $-\frac{3}{2}$. Write an equation of line in slope-intercept form for this line.

A line passes through the point $(6, -7)$ and has a slope of $-\dfrac{5}{2}$. Write an equation of line in slope-intercept form for this line.

Find an equation for the line below. Write your answer in slope-intercept form.



Find an equation for the line that passes through the points $(-4, -4)$ and $(2, 4)$. Write your answer in slope-intercept form.

Write equations for the vertical and horizontal lines passing through the point $(9, -9)$.

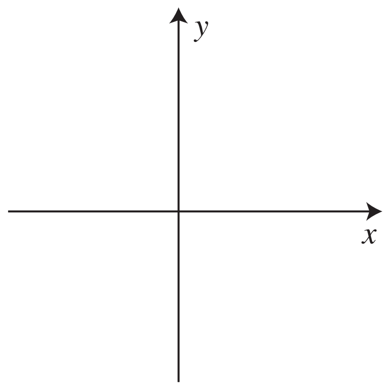
Vertical line: __________

Horizontal line: __________

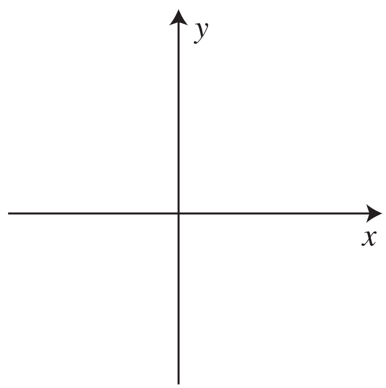
Objective 9d: Graphing Linear Equations

Graph the line.

$y = 2x$

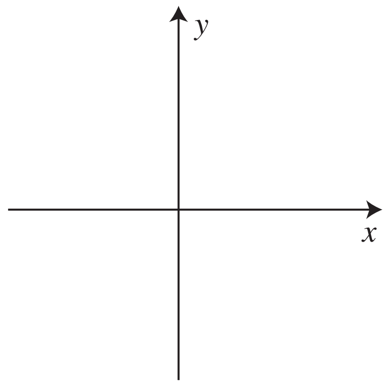


$y = 3x - 7$

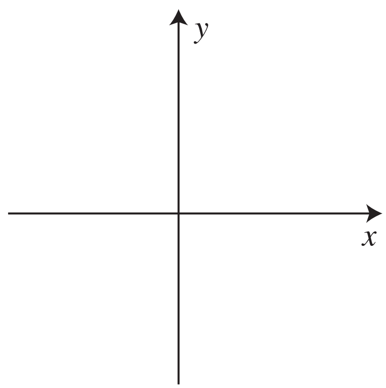


Graph the line.

$$y = -\frac{3}{5}x + 1$$

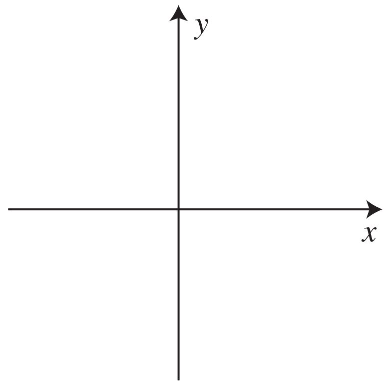


$$-2x + y = -8$$

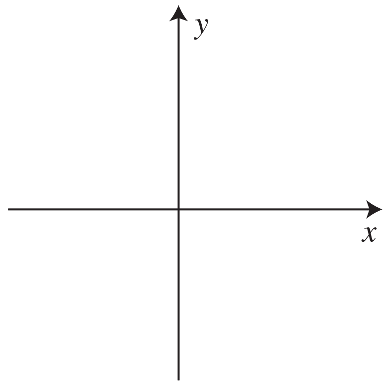


Graph the line.

$y = -5$



$x = 4$



Objective 9e: Parallel and Perpendicular Lines

Slopes of Parallel and Perpendicular Lines

Slopes of Parallel Lines Two nonvertical lines are parallel if and only if their slopes are __________ and they have different y-intercepts. In other words, parallel lines have the same slope.

Slopes of Perpendicular Lines Two nonvertical lines are perpendicular if and only if the product of their slopes is __________. In other words, perpendicular lines have negative reciprocal slopes.

For vertical lines, we must use geometry. Two vertical lines are parallel - they never intersect.

A vertical line and a horizontal are perpendicular - they intersect in a $90°$ angle.

Consider the line $y = -\frac{1}{2}x + 6$**.**

What is the slope of a line parallel to this line?

What is the slope of a line perpendicular to this line?

Consider the line $3x + 9y = -5$**.**

What is the slope of a line parallel to this line?

What is the slope of a line perpendicular to this line?

Consider the line $y = -\frac{3}{2}x + 2$.

a) Find the equation of the line that is perpendicular to this line and passes through the point $(-3, 3)$.

b) Find the equation of the line that is parallel to this line and passes through the point $(-3, 3)$.

Consider the line $y = -\frac{3}{7}x - 2$.

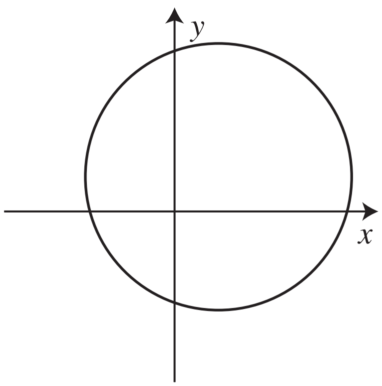
a) Find the equation of the line that is perpendicular to this line and passes through the point $(-7, -2)$.

b) Find the equation of the line that is parallel to this line and passes through the point $(-7, -2)$.

Objective 9f: Circles

A **circle** is a set of points in the xy-plane that are a fixed distance r from a fixed point (h, k). The fixed distance r is called the ______________, and the fixed point (h, k) is called the ______________ of the circle.

The **standard form of an equation of a circle** with radius r and center (h, k) is



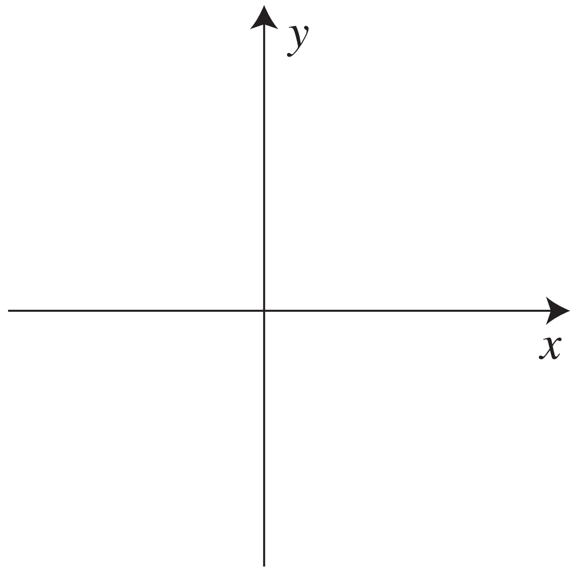
Write an equation of the circle with center $(7, -8)$ and radius 5.

Write an equation of the circle with center $(0, 3)$ and radius 2.

Write an equation of the circle with center $(-8, 4)$ and diameter 12.

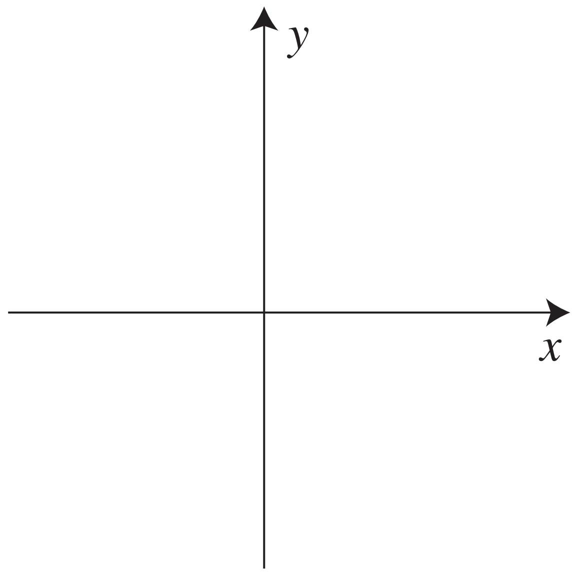
The equation of a circle is given below. Identify the radius and center. Then graph the circle.

$(x+1)^2 + (y-3)^2 = 4$



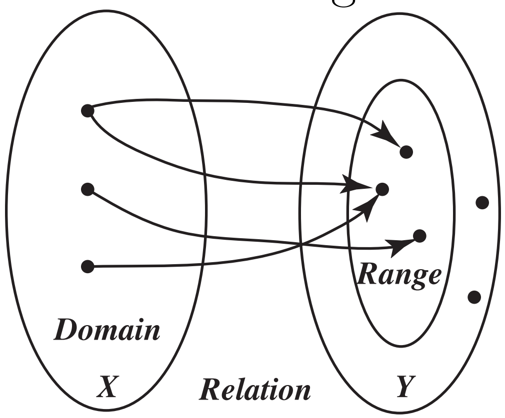
The equation of a circle is given below. Identify the radius and center. Then graph the circle.

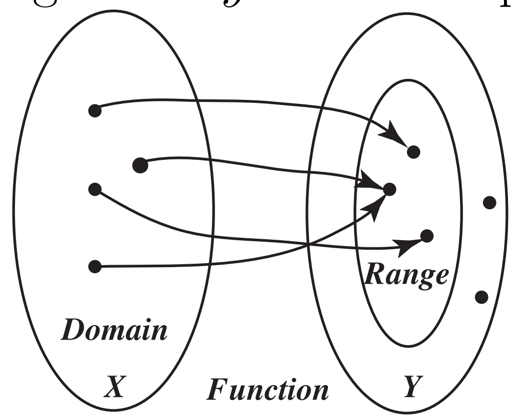
$x^2 + (y - 1)^2 = 1$



Objective 10a: Evaluating Functions

A ______________ is a correspondence between two sets. A relation can be represented as an algebraic rule relating x and y or as a mapping between sets.





One of the most important concepts in mathematics is the **function**. A function is a special type of relation.

Let X and Y be two nonempty sets. A **function** from X into Y is a relation that associates each element of X ______________ element of Y.

The set X is called the _______________. For each element x in X, the corresponding element y in Y is called the **value** of the function at x, or the **image** of x.

The set of all images of the elements in the domain is called the _______________ of the function. In other words, the set of corresponding y-values is the range.

Functions are often denotes by letters such as f, F, g, G, etc. If f is a function, then for each number x in its domain, the corresponding image in the range is designated by the symbol __________, read as __________.

For a function $y = f(x)$, the variable x is called the ________________, and the variable y is called the ________________.

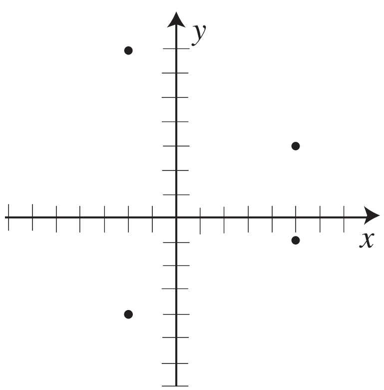
Determine whether each relation represents a function.

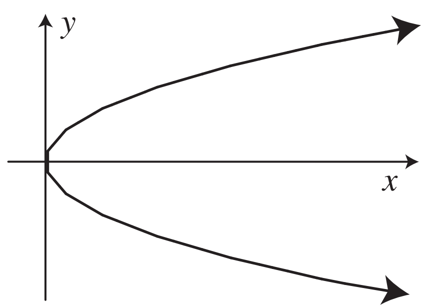
$(-2, 5), (-1, 3), (3, 7), (4, 12)$

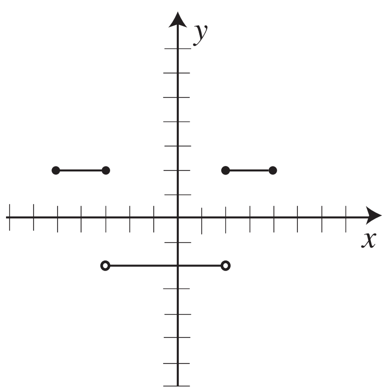
$(-4, 4), (-3, 3), (-2, 2), (-1, 1), (-4, 0)$

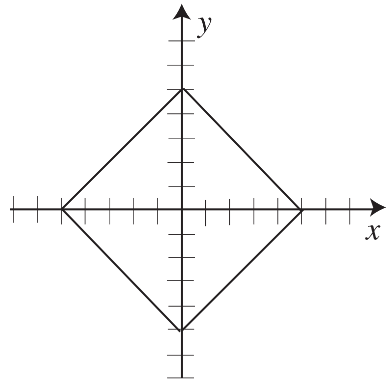
The Vertical Line Test.

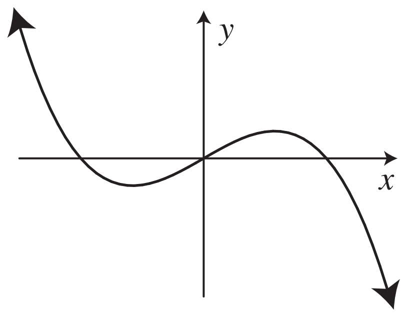
A set of points in the xy-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.











Fill in the values using the function rule given:

$h(x) = 3x + 5$

$x = -1$ $h(x) =$ __________

$x = 0$ $h(x) =$ __________

$x = 1$ $h(x) =$ __________

$x = 5$ $h(x) =$ __________

Find the following for each function:

The functions f and g are defined as follows.

$f(x) = 3x^2 - 3x$ and $g(x) = -2x - 3$

Find $f(-4)$ and $g(7)$. Simplify your answer as much as possible.

The function h is defined as follows.

$$h(x) = \frac{4x}{2x - 17}$$

Find $h(6)$. Simplify your answer as much as possible.

The function g is defined as follows.

$$g(x) = \frac{x^2 - 5x - 50}{x - 1}$$

Find $g(9)$. Simplify your answer as much as possible.

The function f is defined as follows.

$$f(x) = \mid 12x - 17 \mid$$

Find $f(-\frac{1}{4})$. Simplify your answer as much as possible.

The function f is defined as follows.

$$f(x) = \frac{x+4}{3x+3}$$

Find $f(5z)$.

The function f is defined as follows.

$$f(x) = 5x^2 - 3$$

Find $f(x+1)$. Write your answer without parentheses, and simplify it as much as possible.

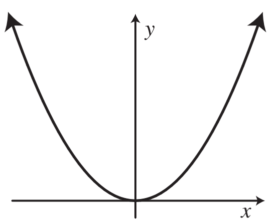
Objective 10b: Even and Odd Functions

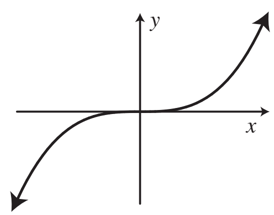
A function f is **even** if, for every number x in its domain, the number $-x$ is also in the domain and ____________________.

A function is even if and only if its graph is symmetric with respect to the _____________.

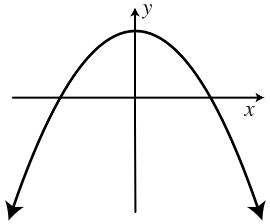
A function f is **odd** if, for every number x in its domain, the number $-x$ is also in the domain and ____________________.

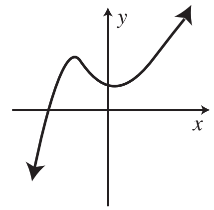
A function is odd if and only if its graph is symmetric with respect to the _____________.

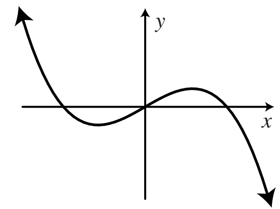


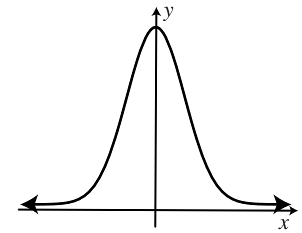


The graph of a function is given. Determine whether each function is even, odd, or neither.









To determine **algebraically** whether a function is even, odd, or neither, find $f(-x)$.

If $f(-x) =$ ________, the function is ________

If $f(-x) =$ ________, the function is ________

If $f(-x) =$ ________________, the function is ________________

Determine algebraically whether each function is even, odd, or neither.

$f(x) = 2x^4 - x^2 + 3$

$f(x) = 3x^3 + 5$

Determine algebraically whether each function is even, odd, or neither.

$f(x) = 3x^3 - x$

$f(x) = \dfrac{x^4}{x^2 + 1}$

Determine algebraically whether each function is even, odd, or neither.

$$f(x) = \frac{-2x^3}{x^2+1}$$

$$f(x) = \frac{x^2+1}{x^5}$$

Objective 10c: Domain of Functions

Note:

It is important in this objective and going forward to be able to identify differences in functions.

Polynomial Functions: all powers have non-negative integers. (i.e. 0, 1, 2,...)

$f(x) = x^3 + 2x - 5$ $\qquad$ $f(x) = x^3 + \sqrt{x}$

Rational Functions: must have form $\frac{p}{q}$ where p and q are polynomials and the degree of $q \geq 1$

$f(x) = \dfrac{x^2+5}{x^2-1}$ $\qquad$ $f(x) = \dfrac{\sqrt{x}+3}{2x}$ $\qquad$ $f(x) = \dfrac{x^2-7}{2}$ $\qquad$ $f(x) = \dfrac{1}{3x+2}$

Radical Functions: any powers are rational integers. (i.e. $\frac{1}{2}, \frac{1}{3}, \ldots$)

$f(x) = x^3 + \sqrt{x} - 3$ $\qquad$ $f(x) = (3x+2)^{1/3}$ $\qquad$ $f(x) = 2x^2 - x^{2/3}$

Finding the Domain of a Function.

What is the largest set of real numbers that can be substituted in for x? When would you *not* be able to substitute a particular value in for x?

1. Start with the domain as the set of all real numbers.

2. If the equation has a **denominator**, exclude any numbers that give a zero denominator. In other words, set the denominator $\neq 0$ and solve.

3. If the equation has a **radical of even index**, exclude any numbers that cause the expression inside the radical (the radicand) to be negative. In other words, set the radicand ≥ 0 and solve.

Note: If the function is not a rational function with a variable in the denominator or a radical function with a variable under the radical, the domain is automatically all real numbers.

Suppose that the relation T is defined as follows:

$T = \{(7, -4), (-3, -4), (6, 7)\}$

Give the domain and range of T. Write your answers using set notation.

The functions f and g are defined as follows.

$$f(x) = \frac{x}{x^2 + 36} \qquad g(x) = \frac{x+1}{x^2 - 1}$$

For each function, find the domain. Write each answer as an interval or union of intervals.

Find the domain of the function.

$g(x) = \sqrt{x - 3}$

Write your answer using interval notation.

Find the domain of the function.

$f(x) = \sqrt{3 - x}$

Write your answer using interval notation.

Find the domain of the function.

$$f(x) = \frac{4}{\sqrt{x-9}}$$

Write your answer using interval notation.

Find the domain of the function.

$$f(x) = \frac{\sqrt{-7+x}}{9-x}$$

Write your answer using interval notation.

Find the domain of the function.

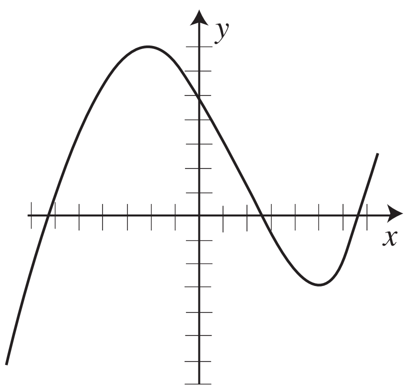
$$f(x) = \sqrt{\frac{17}{x-7}}$$

Write your answer using interval notation.

Objective 10d: Information from Graphs

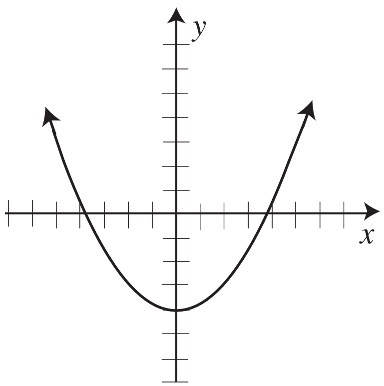
If (x, y) is a point on the graph of a function f, then y is the value of f at x; that is, $y = f(x)$.

The graph of a function f is shown below.



Find $f(-2)$.

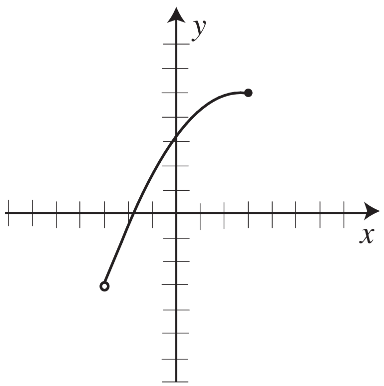
The graph of a function f is shown below.



Find one value of x for which $f(x) = 3$ and find $f(2)$.

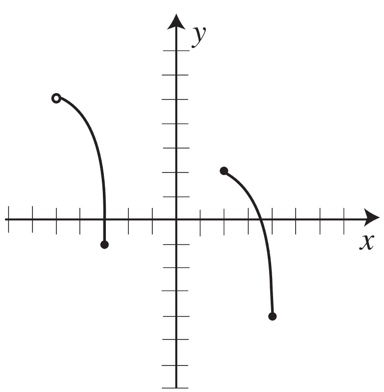
The entire graph of the function f is shown in the figure below.

Write the domain and range of f using interval notation.



The entire graph of the function g is shown in the figure below.

Write the domain and range of g as intervals or unions of intervals.



Use a Graph to Determine Where a Function is Increasing, Decreasing, or Constant

A function f is **increasing** on an open interval I if, for any x_1 and x_2 in I, with $x_1 < x_2$, ______________

That is, as x-values get larger, the values of the function also get _________.

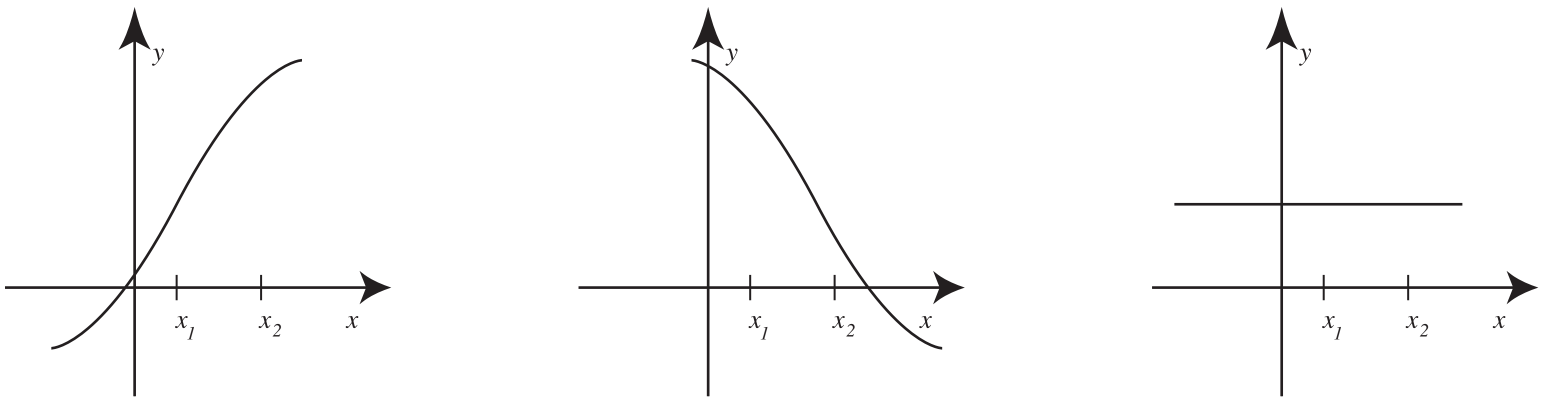
A function f is **decreasing** on an open interval I if, for any x_1 and x_2 in I, with $x_1 < x_2$, ______________

That is, as x-values get larger, the values of the function get _________.

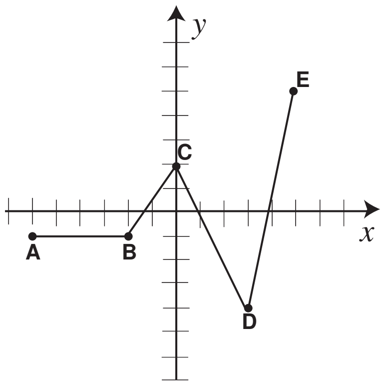
A function f is **constant** on an open interval I if, for any x in I, the values $f(x)$ are ______________

That is, as the x-values get larger, the values of the function _____________.

Note: You will be asked to give the intervasl(s) **on which** the function is increasing, decreasing, or constant. That is, you will answer _______________.

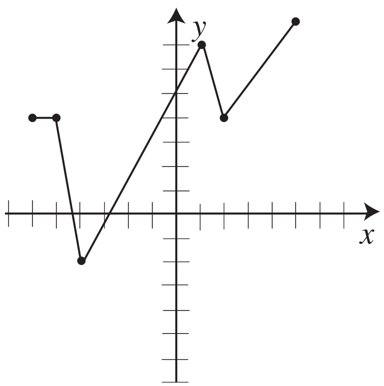


Determine the interval(s) on which the function is (strictly) decreasing.



Determine the interval(s) on which the function is (strictly) increasing.

Write your answer as an interval or list of intervals. When writing a list of intervals, make sure to separate each interval with a comma and to use as few intervals as possible.



Absolute Maximum/Minimum, Local Maximum/Minimum.

Let f be a function defined on some interval I.

A function f has a ____________________ at c if there is an *open* interval in I containing c so that, for all x in this open interval, we have ______________. We call $f(c)$ a **local maximum value of** f.

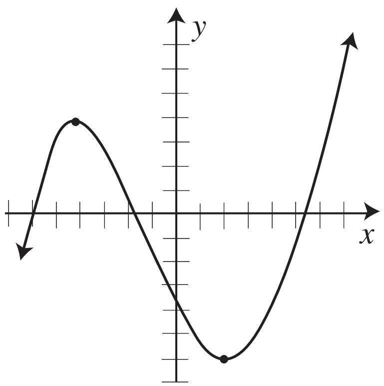
A function f has a ____________________ at c if there is an *open* interval in I containing c so that, for all x in this open interval, we have ______________. We call $f(c)$ a **local minimum value of f**.

If there is a number u in I for which ______________ for all x in I, the f has an ____________________ and the number $f(u)$ is the **absolute maximum of** f **on** I.

If there is a number v in I for which ______________ for all x in I, the f has an ____________________ and the number $f(v)$ is the **absolute minimum of** f **on** I.

Note: An absolute max/min may occur at an endpoint of the interval I.

Here is a graph of the function h.



Use the graph to find the following. If there is more than one answer, separate them with commas.

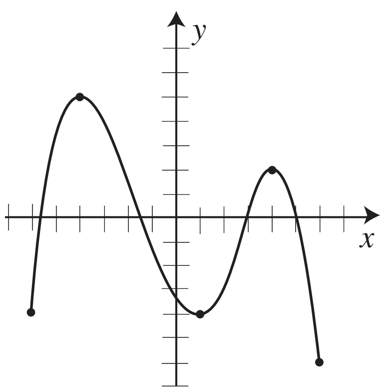
a) All values at which h has a local minimum:

b) All local minimum values of h:

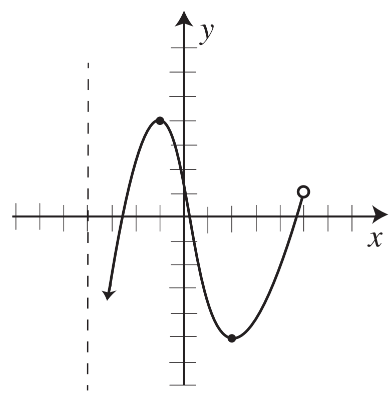
The graphs of the functions g and h are shown below. For each group, find the absolute maximum and absolute minimum.

Assume that the dashed line is a vertical asymptote that the graph does not cross.

Graph of g:



Graph of h:



Absolute maximum of g:

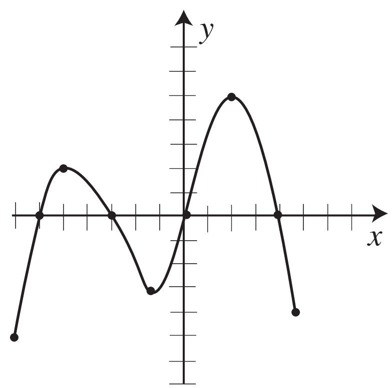
Absolute maximum of h:

Absolute minimum of g:

Absolute minimum of h:

More Graphs of Functions.

Use the graph of the function $y = f(x)$ below to answer the questions.

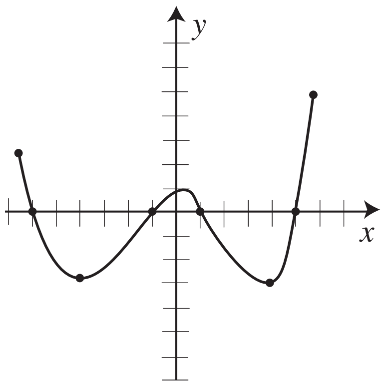


a) Is $f(1)$ positive?

b) For which values(s) of x is $f(x) > 0$? Write your answer using interval notation.

c) For which value(s) of x is $f(x) = 0$? If there is more than one value, separate them with commas.

Use the graph of the function $y = f(x)$ below to answer the questions.



a) Is $f(-2)$ positive?

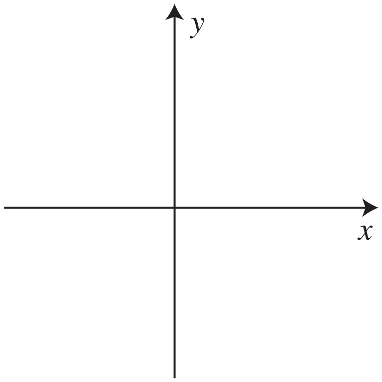
b) For which values(s) of x is $f(x) < 0$? Write your answer using interval notation.

c) For which value(s) of x is $f(x) = 0$? If there is more than one value, separate them with commas.

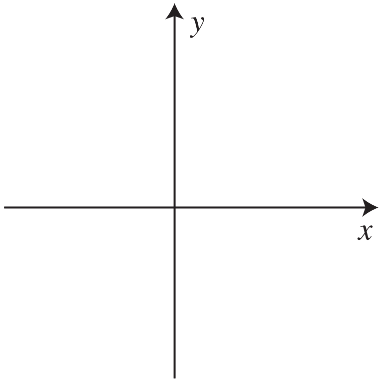
Objective 10e: Parent Functions and Transformations

For each of these 8 functions, you should know the graph and the properties.

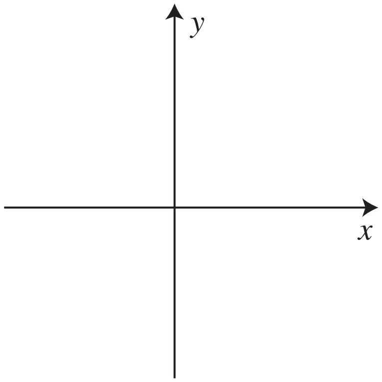
1) **Constant Function** $f(x) = b$, b is a real number.



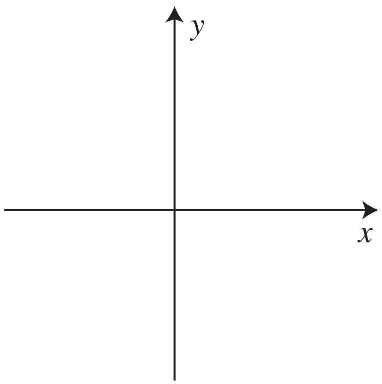
2) **Identity Function** $f(x) = x$.



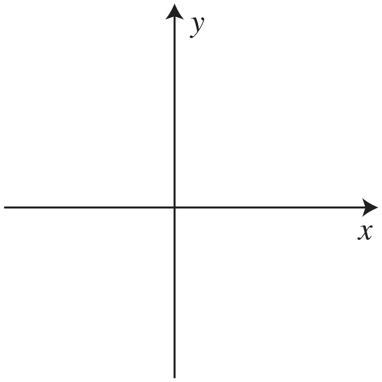
3) **Squaring function** $f(x) = x^2$.



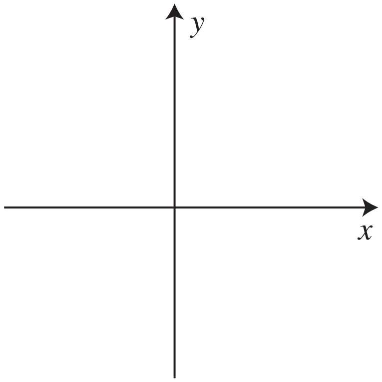
4) **Cubing function** $f(x) = x^3$.



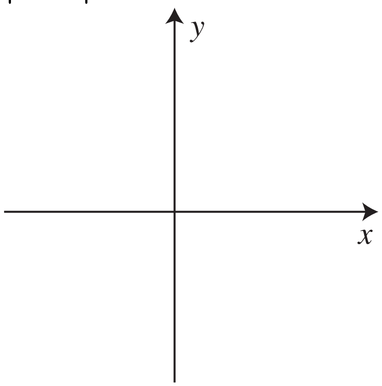
5) **Square Root function** $f(x) = \sqrt{x}$.



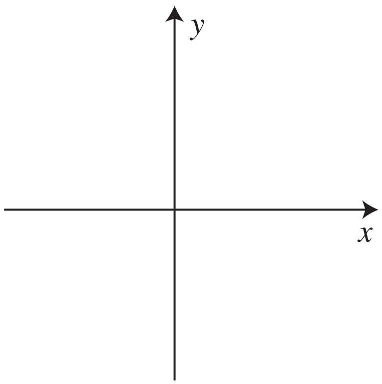
6) **Cube Root function** $f(x) = \sqrt[3]{x}$.



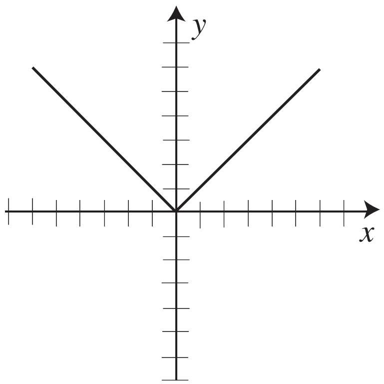
7) **Absolute value function** $f(x) = |\ x\ |$.



8) **Reciprocal function** $f(x) = \dfrac{1}{x}$.

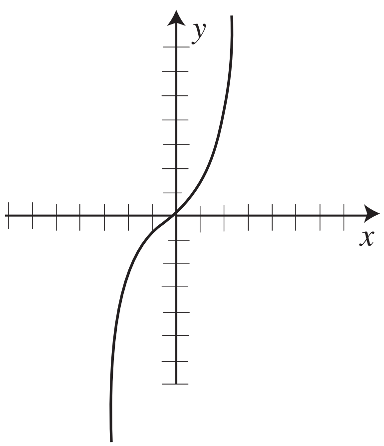


For each graph, choose the function that best describes it.



$f(x) = 1$ $\quad f(x) = x$ $\quad f(x) = x^2$ $\quad f(x) = x^3$

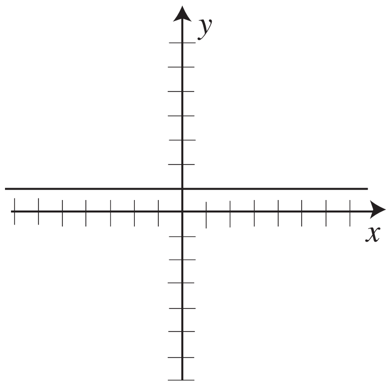
$f(x) = \sqrt{x}$ $\quad f(x) = | x |$ $\quad f(x) = \dfrac{1}{x}$ $\quad f(x) = \dfrac{1}{x^2}$



$f(x) = 1$ $\quad f(x) = x$ $\quad f(x) = x^2$ $\quad f(x) = x^3$

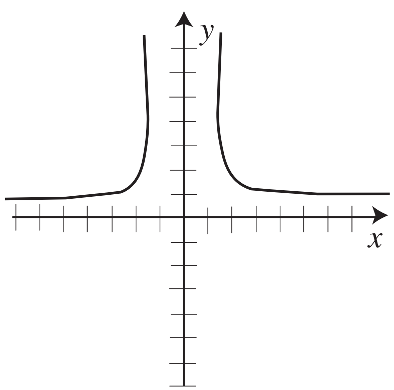
$f(x) = \sqrt{x}$ $\quad f(x) = | x |$ $\quad f(x) = \dfrac{1}{x}$ $\quad f(x) = \dfrac{1}{x^2}$

For each graph, choose the function that best describes it.



$f(x) = 1$ $\quad$ $f(x) = x$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = x^3$

$f(x) = \sqrt{x}$ $\quad$ $f(x) = \mid x \mid$ $\quad$ $f(x) = \dfrac{1}{x}$ $\quad$ $f(x) = \dfrac{1}{x^2}$



$f(x) = 1$ $\quad$ $f(x) = x$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = x^3$

$f(x) = \sqrt{x}$ $\quad$ $f(x) = \mid x \mid$ $\quad$ $f(x) = \dfrac{1}{x}$ $\quad$ $f(x) = \dfrac{1}{x^2}$

Graphing Techniques: Transformations

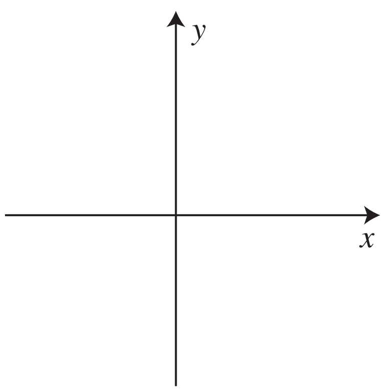
Graph Functions using Vertical and Horizontal Shifts

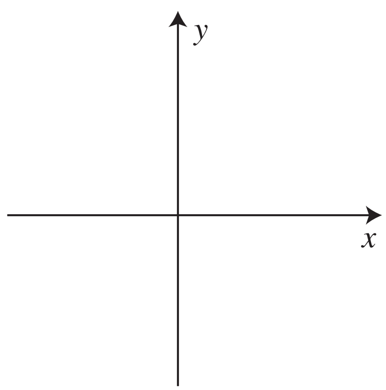
How do these compare?

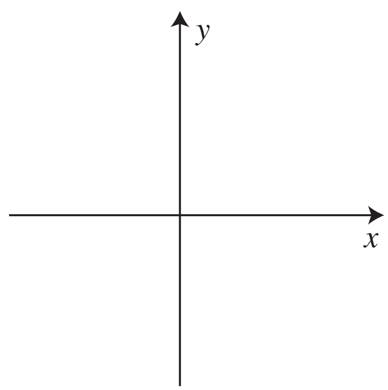
$y = f(x)$ $\qquad$ $y = f(x) + c, c > 0$ $\qquad$ $y = f(x) - c, c > 0$

We will consider a specific example to justify the general case.

$y = \sqrt{x}$ $\qquad$ $y = \sqrt{x} + 3$ $\qquad$ $y = \sqrt{x} - 3$





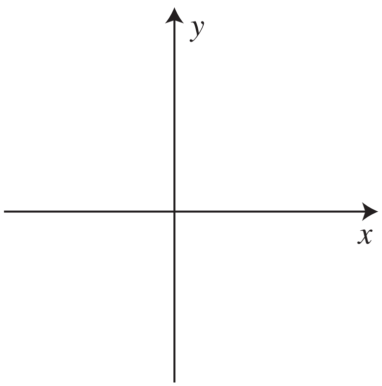


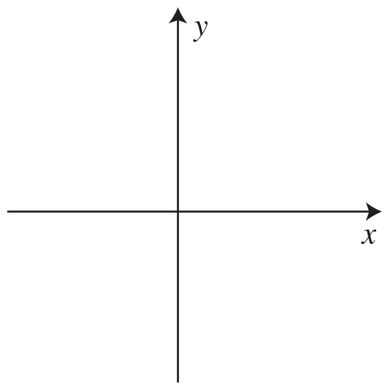
How do these compare?

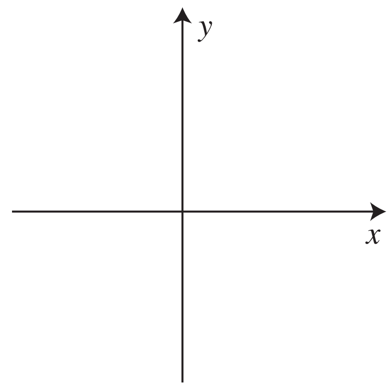
$y = f(x)$ $\quad$ $y = f(x + c), c > 0$ $\quad$ $y = f(x - c), c > 0$

We will consider a specific example to justify the general case.

$y = \sqrt{x}$ $\quad$ $y = \sqrt{x + 3}$ $\quad$ $y = \sqrt{x - 3}$







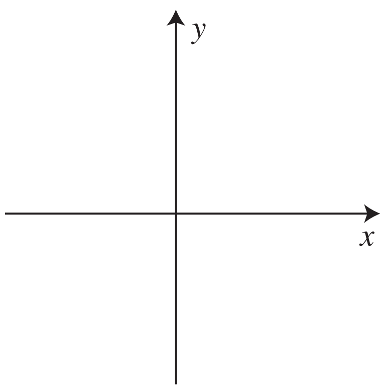
Graph Functions using Reflections about the x-axis and the y-axis

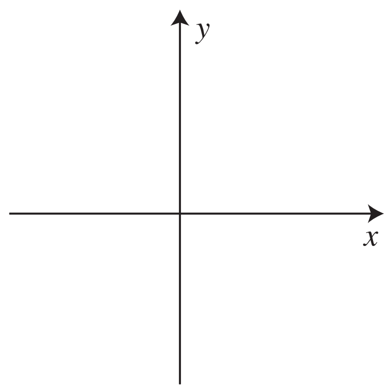
How do these compare?

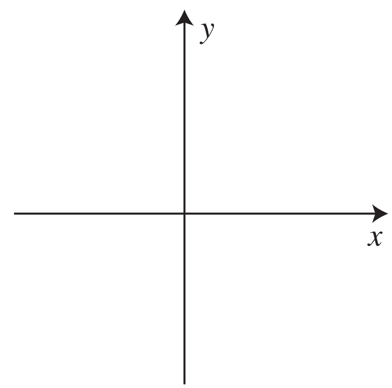
$y = f(x)$ $y = -f(x)$ $y = f(-x)$

We will consider a specific example to justify the general case.

$y = \sqrt{x}$ $y = -\sqrt{x}$ $y = \sqrt{-x}$







Summary of Graphing Techniques

To Graph: **Draw the Graph of f and:**

Vertical shifts

$y = f(x) + k, k > 0$

$y = f(x) - k, k > 0$

Horizontal shifts

$y = f(x + h), h > 0$

$y = f(x - h), h > 0$

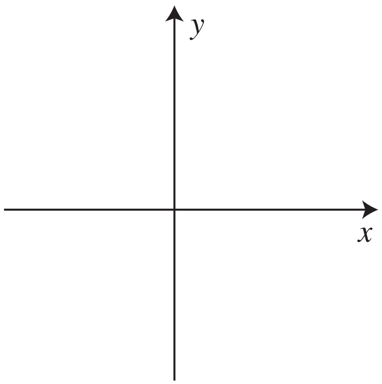
Reflections

$y = -f(x)$

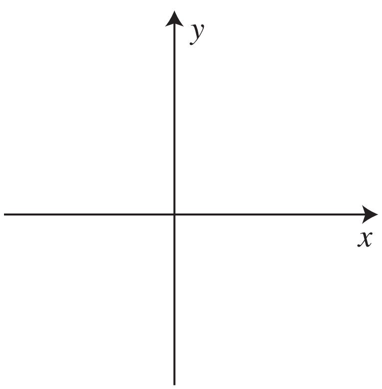
$y = f(-x)$

Translate each graph as specified below.

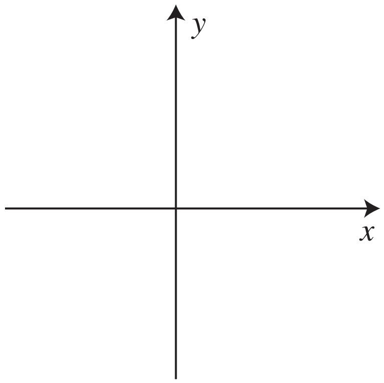
a) Translate the graph of $y = x^2$ to get the graph of $y = x^2 - 5$.



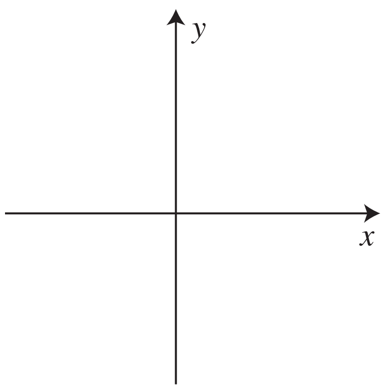
b) Translate the graph of $y = x^2$ to get the graph of $y = (x - 2)^2$.



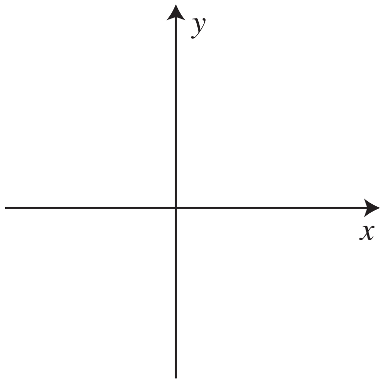
c) Translate the graph of $y = x^2$ to get the graph of $y = (x+1)^2 + 5$.



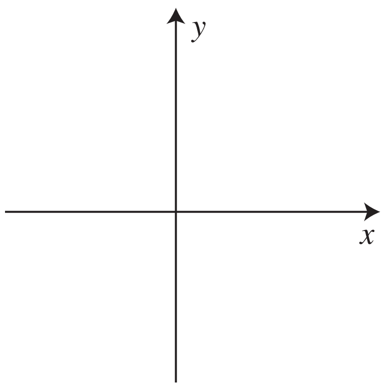
d) Translate the graph of $y =| x |$ to get the graph of $y =| x | +3$.



e) Translate the graph of $y = | x |$ to get the graph of $y = | x + 5 |$.

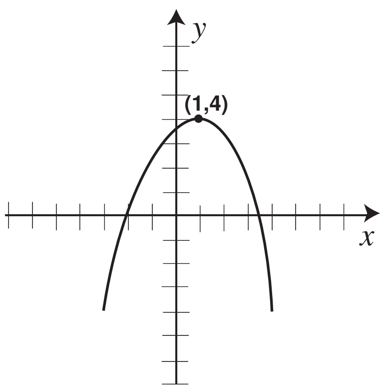


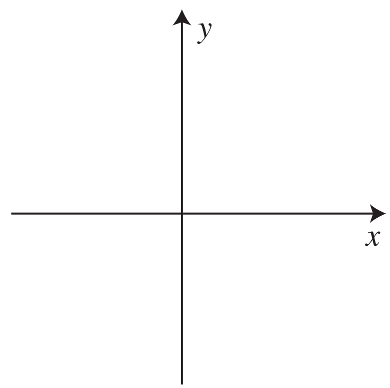
f) Translate the graph of $y = | x |$ to get the graph of $y = | x - 3 | + 5$.



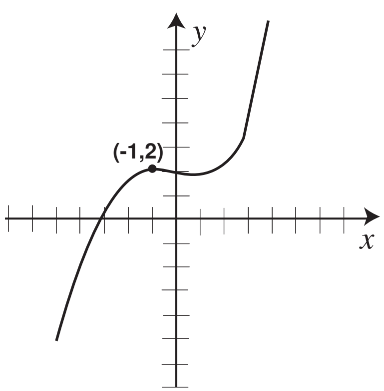
Translate each graph as specified below.

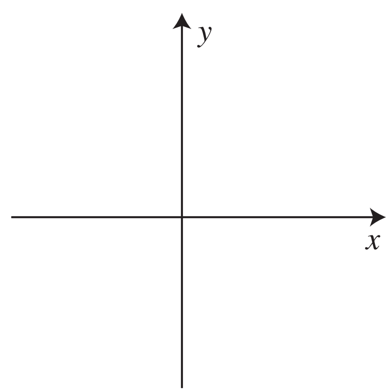
a) The graph of $y = f(x)$ is given below. Translate it to get the graph of $y = f(x) - 4$.





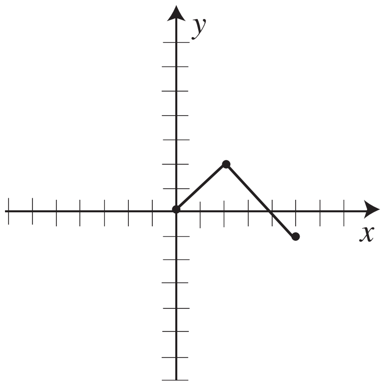
b) The graph of $y = g(x)$ is given below. Translate it to get the graph of $y = g(x+2)$.

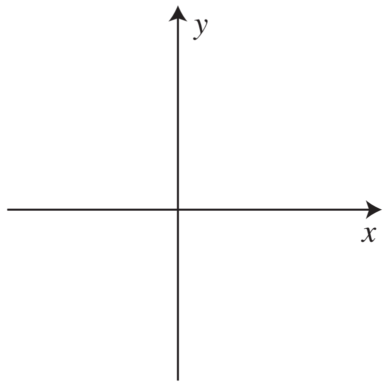




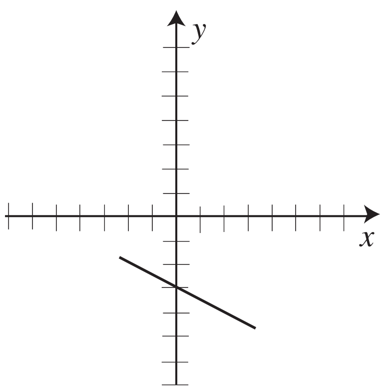
Translate each graph as specified below.

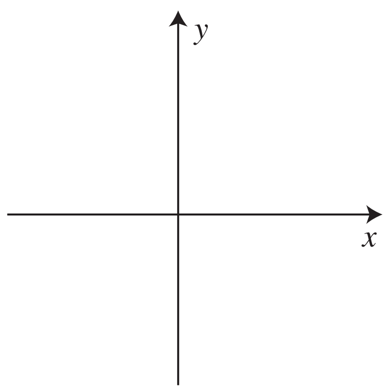
a) The graph of $y = f(x)$ is given below. Translate it to get the graph of $y = -f(x)$.





b) The graph of $y = g(x)$ is given below. Translate it to get the graph of $y = g(-x)$.





Objective 10f: Piecewise-Defined Functions

A piecewise-defined function has a different function rule for different parts of the domain.

To evaluate a piecewise-defined function, compare the x-value (input value) with the restrictions on the right - that tells you which ______________ function rule to use.

Suppose the function f is defined, for all real numbers, as follows.

$$f(x) = \begin{cases} \frac{1}{4}x - 1 & \text{if } x \neq -2 \\ 2 & \text{if } x = -2 \end{cases}$$

Find $f(-5), f(-2), f(1)$.

Suppose the function f is defined, for all real numbers, as follows.

$$f(x) = \begin{cases} -4 & \text{if } x < -2 \\ (x+1)^2 - 2 & \text{if } -2 \leq x < 1 \\ -\frac{1}{4}x + 1 & \text{if } x \geq 1 \end{cases}$$

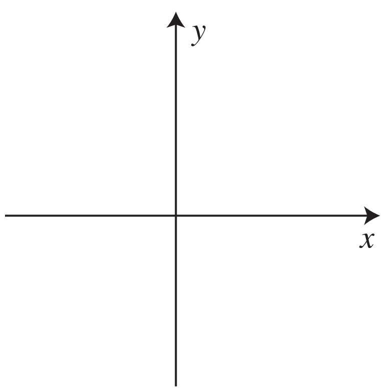
Find $f(0), f(1), f(5)$.

When graphing a piecewise-defined function, graph each "piece" on the specified domain.

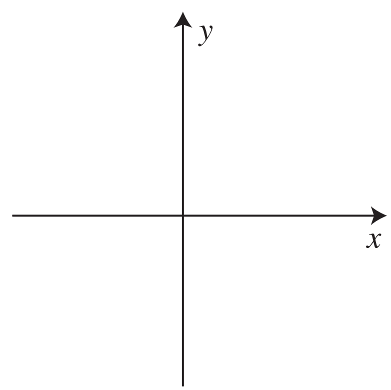
A warm-up question:

Recall: Graph a linear inequality with one variable. That is, graph $x > 1$. Note what value you plot and the different notation you use depending on the inequality sign.

Graph $f(x) = x - 2$ if $x \geq 1$



Graph $f(x) = x - 2$ if $x > 1$



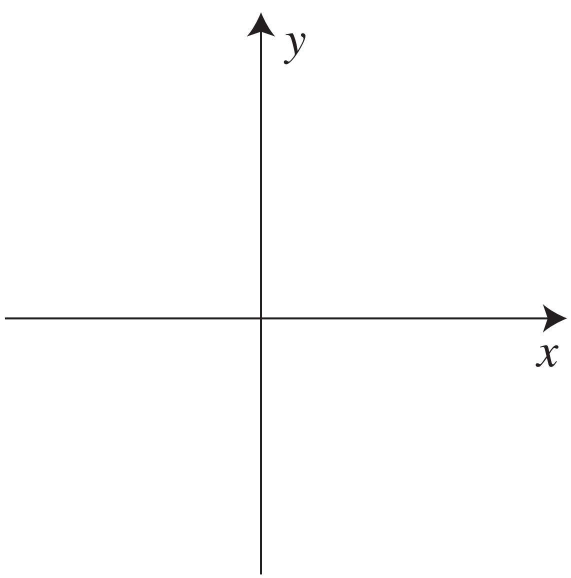
When graphing a piecewise-defined function, graph each "piece" on the specified domain.

Note: At the breaking points of the domain, we draw open circles to indicate where the breaking point is not included because of a strict inequality; we draw a closed circle where the breaking point is included because of a non-strict inequality. That is, breaking points are plotted for each "piece" using different notation.

Suppose the function f is defined, for all real numbers, as follows.

$$f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ -2x-3 & \text{if } x \geq -2 \end{cases}$$

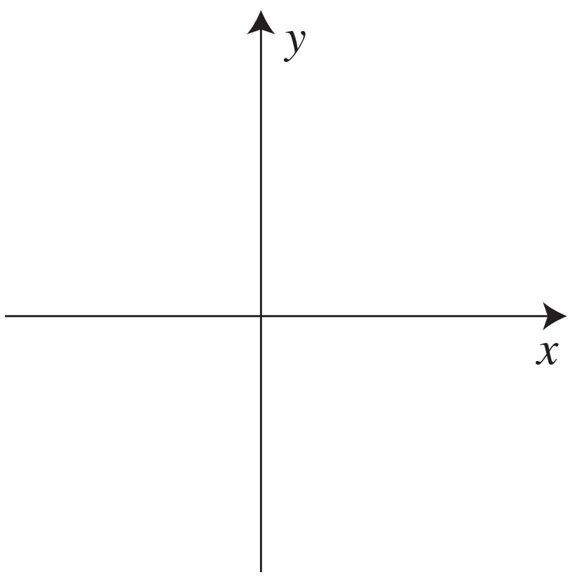
Graph the function f. Then determine whether or not the function is continuous.



Suppose the function f is defined, for all real numbers, as follows.

$$f(x) = \begin{cases} x+6 & \text{if } -5 \le x < 1 \\ 9 & \text{if } x = 1 \\ -x+4 & \text{if } x > 1 \end{cases}$$

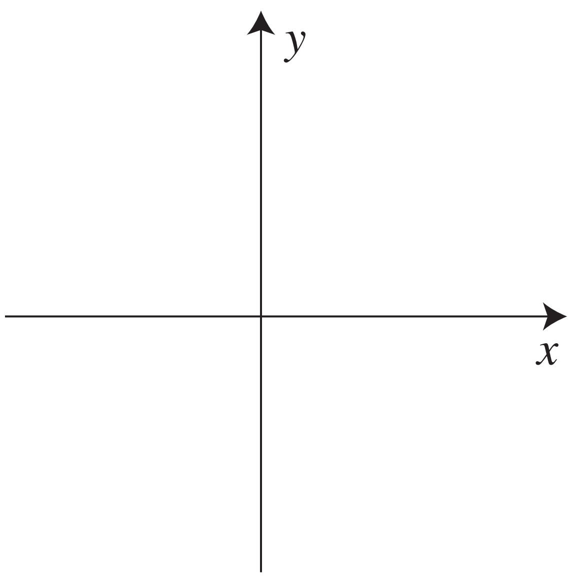
Graph the function f. Then determine whether or not the function is continuous.



Suppose the function f is defined, for all real numbers, as follows.

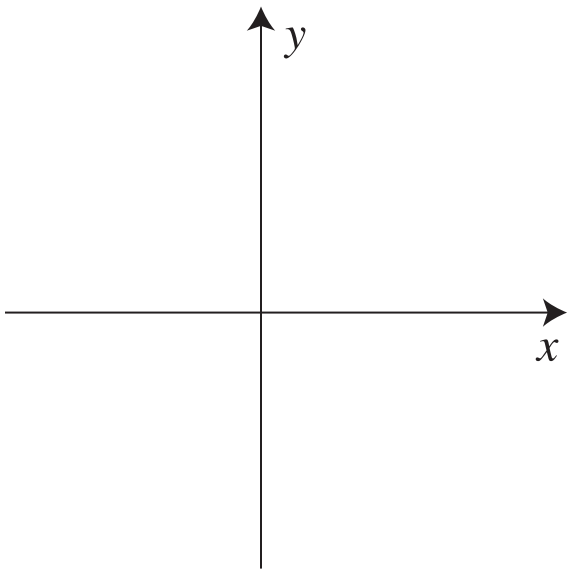
$$f(x) = \begin{cases} 1 - x^2 & \text{if } x < 1 \\ -2x - 3 & \text{if } x \geq 1 \end{cases}$$

Graph the function f. Then determine whether or not the function is continuous.



Suppose the function f is defined, for all real numbers, as follows.

Graph $f(x) = \begin{cases} x - 2 & \text{if } x > 1 \\ x & \text{if } x \leq 1 \end{cases}$



Objective 10g: Linear Function Applications

A construction crew is lengthening a road that originally measured 55 miles. The crew is adding one mile to the road each day. The length, L (in miles), after d days of construction is given by the following function.

$L(d) = 55 + d$

What is the length of the road after 32 days?

A printing service charges a set-up fee of 11.00 for each order and 9 cents more for each copy. The total cost, C (in dollars), for an order of x copies is given by the following fuction.

$C(x) = 11.00 + 0.09x$

What is the total cost for an order of 30 copies?

A construction crew needs to pave a road that is 201 miles long. The crew paves 8 miles of the road each day. The length, L (in miles), that is left to be paved after d days is given by the following function.

$L(d) = 201 - 8d$

Answer the following questions:

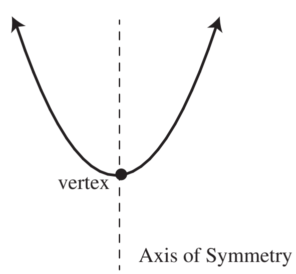
a) How many miles of the road does the crew have left to pave after 13 days?

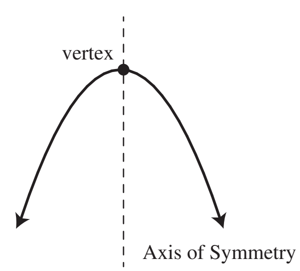
b) If 129 miles of the road is left to be paved, how many days has the crew been paving the road?

Objective 11: Quadratic Functions

A **quadratic function** is a function of the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$. We already know the domain of a quadratic function is ______________

The graph of a quadratic function is a **parabola**. The lowest or highest point of a parabola is called the ______________. The vertical line passing through the vertex is called the ____________________.





Standard Form

For a quadratic function in the form $f(x) = ax^2 + bx + c$,

vertex $= \left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$ and the

Axis of Symmetry is the vertical line $x = -\dfrac{b}{2a}$.

The parabola opens up if ________________; the vertex is a minimum point and $f\left(-\dfrac{b}{2a}\right)$ is the **minimum value** of f.

The parabola opens down if ________________; the vertex is a maximum point and $f\left(-\dfrac{b}{2a}\right)$ is the **maximum value** of f.

The y-intercept is the value of f at $x = 0$. That is, the y-intercept is ______________

The x-intercepts, if any, are found by letting $y = 0$. That is, the x-intercepts, if any, are found by solving the quadratic equation $ax^2 + bx + c = 0$.

Recall the **Quadratic Formula**. $x =$

If the discriminant $b^2 - 4ac > 0$, the graph has ____________________

If the discriminant $b^2 - 4ac = 0$, the graph has ____________________

If the discriminant $b^2 - 4ac = 0$, the graph will touch the x-axis at its ________________

If the discriminant $b^2 - 4ac < 0$, the graph has ____________________

(h,k) Form

When the quadratic function is in the form: $f(x) = a(x-h)^2+k$, we can use what we learned about **transformations** to identify the vertex.

h is the ________________ shift and k is the ________________ shift.

The **vertex** is ____________.

In this form, it is still true that if $a > 0$ the parabola opens _________ and if $a < 0$ the parabola opens _________.

The y-intercept is the value of f at $x = 0$. That is, the y-intercept $= f(0)$. You must calculate it.

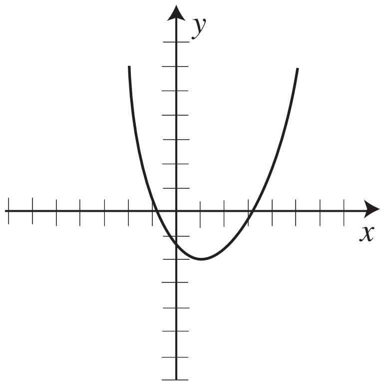
The x-intercepts, if any, are found by letting $y = 0$. That is, the x-intercepts, if any, are found by solving the quadratic equation $a(x-h)^2 + k = 0$.

This equation is best solved by the **Square-root Property**. Do not ________________

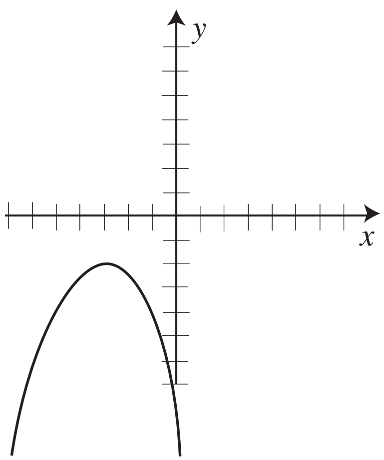
Summary

	Standard Form	(h, k) Form
	$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$
vertex	$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	(h, k)
a	$a > 0$ opens up $a < 0$ opens down	same same
y-int	let $x = 0$, solve for y y-int=c	same calculate the y-intercept
x-int(s)	let $y = 0$, solve for x (factor or quad. formula)	same (use square-root property)

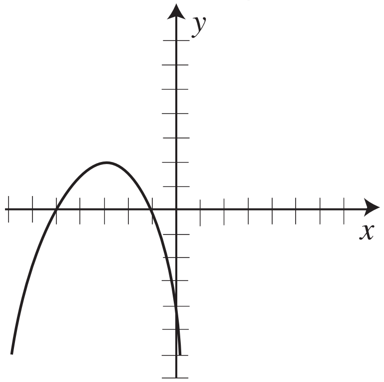
The graph of a quadratic function with vertex $(1, -2)$ is shown in the figure below. Find the domain and range.



The graph of a quadratic function with vertex $(-3, -2)$ is shown in the figure below. Find the domain and range.



Use the graph of the parabola to answer the following questions.



a) Does the parabola open upward or downward?

b) Find the intercepts.

- x-intercepts:

- y-intercepts:

c) Find the coordinates of the vertex.

d) Find the equation of the axis of symmetry.

Find the x-intercept(s) and coordinates of the vertex for the given parabola. If there is more than one x-intercept, separate them with commas.

$y = x^2 - 2x - 15$

Answer the questions below about the quadratic function.

$g(x) = 2x^2 + 4x + 6$

a) Does the function have a minimum or maximum value?

b) Where is the function's minimum or maximum value?

c) What is the minimum or maximum value?

A medical equipment industry manufactures X-ray machines. The unit cost C (the cost in dollars to make each X-ray machine) depends on the number of machines made. If x machines are made, then the unit cost is given by the function $C(x) = 0.5x^2 - 280x + 58,665$. What is the minimum unit cost? Do not round your answer.

A ball is thrown vertically upward. After t seconds, its height h (in feet) is given by the function $h(t) = 112t - 16t^2$. After how long will it reach its maximum height? Do not round your answer.

Find the range of the quadratic function.

$g(x) = x^2 - 4x + 6$

Objective 12: Polynomial and Rational Inequalities

When you are asked to solve a polynomial or radical inequality, you are looking for the solutions for x that satisfy the inequality statement.

Introduction. Why a special technique is needed.

If $a \cdot b = 0$, then it must be true that __________ or __________.

However, if $a \cdot b > 0$, then it does not have to be true that __________ or __________. In fact, both numbers could be negative (< 0).

It would require 2 cases to express this solution:
Case 1. __________ and __________ Or Case 2. __________ *and* __________

If we consider $a \cdot b \cdot c > 0$ (three numbers multiplied with the result being positive), then it would require 4 cases to describe this solution, and in only one case is it true that $a > 0$ and $b > 0$ and $c > 0$.

We need a process that doesn't require determining, and then analyzing, cases.

The process is based on the Factor Theorem and the Intermediate Value Theorem.

Steps for Solving Polynomial and Rational Inequalities Algebraically

Step 1 Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:
$f(x) > 0$, $f(x) \geq 0$, $f(x) < 0$, $f(x) \leq 0$.

For **rational expressions**, be sure that the left side is written as a **single quotient**, and **find the domain of** f.

Step 2 Factor, if needed.

Step 3 Find Partitioning Points. For a polynomial inequality, these are values that make the expression ____________. In the case of a rational inequality, these are values that make the expression ____________(from factors in the numerator) or make the expression ______________ (from factors in the denominator).

Set each factor equal to 0 **to find these Partitioning Points.**

Step 4 Mark partitioning points on a number line to separate the real number line into intervals.

Step 5 Make a Sign-Chart. Select a value in each interval that's created by these partitioning points (don't use one of end-points). Plug this value into each separate factor and record whether the result is $+$ or $-$. Consolidate the signs from all the factors.

Note 1:

(a) If the value of f is positive, then $f(x) > 0$ for all numbers x in the interval.

(b) If the value of f is negative, then $f(x) < 0$ for all numbers x in the interval.

Notation Rules:

For **polynomial inequalities**, notation follows inequality sign. So,

$<, >$ is ______________________ and $\leq, \geq$ is ______________________

For **rational inequalities**:

notation for partitioning points from the **numerator** follow inequality sign

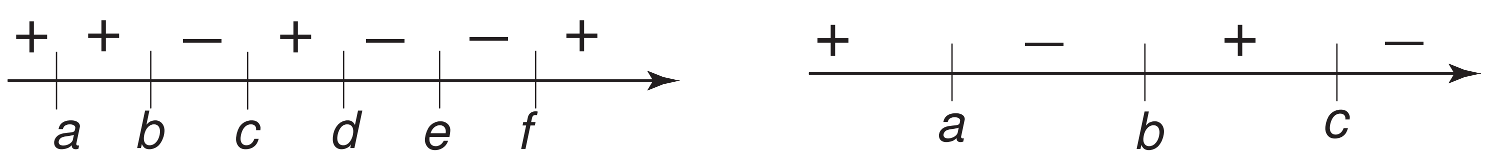
notation for partitioning points from the **denominator** is always open (parentheses).

Note 2:

(a) In the factored form, if any **factor(s)** is/are raised to an ________ power, then the sign-chart will not ____________ (In each case, the associated partitioning point has even-multiplicity and the sign-chart will not alternate at each of these partitioning points.)

For example: x^4 is a factor raised to an even power. $(x+1)^2$ is a factor raised to an even power. (x^2+1) is **not** a factor squared.

(b) In the factored form, if any **factor(s)** is/are **not** raised to an ________ power, then the sign-chart will ____________



Solve the following inequality. Write your answer as an interval or union of intervals.

$(x-5)(x-2)(x+7) \leq 0$

$(5-x)(x+2)^2 > 0$

Solve the following inequality. Write your answer as an interval or union of intervals.

$(4-x)(x+3)(x^2+1) \leq 0$

$x^3 - 12x \leq x^2$

Solve the following inequality. Write your answer as an interval or union of intervals.

$$\frac{x-1}{x+4} \geq 0$$

$$\frac{(x+5)^2}{x^2-4} \geq 0$$

Solve the following inequality. Write your answer as an interval or union of intervals.

$$\frac{15}{x-7} \geq 5$$

Solve the following inequality. Write your answer as an interval or union of intervals.

$$\frac{12}{x-3} < 4$$

Objective 13: Combining Functions

Just like we can add, subtract, multiply, and divide numbers and algebraic expressions, we can also add, subtract, multiply, and divide functions.

Sum, Difference, Product, or Quotient of Two Functions

The **sum** $f + g$ is defined by $(f + g)(x) =$ ________

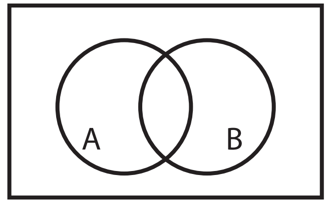
The **difference** $f - g$ is defined by $(f - g)(x) =$ ________

The **product** $f \cdot g$ is defined by $(f \cdot g)(x) =$ ________

The **quotient** $\frac{f}{g}$ is defined by $\left(\frac{f}{g}\right)(x) =$ ________

Note: The **domain** of the sum, difference, or product of two functions consists of all numbers x in the domain of f **and** g.

Note: The **domain** of the quotient of two functions, $\frac{f}{g}$, consists of all numbers x such that $g(x) \neq 0$ and that are in the domain of f **and** g.

"And" means: 

Suppose the functions r and s are defined for all real numbers x as follows.

$r(x) = x + 3$ $\quad\quad$ $s(x) = 2x + 2$

Write expressions for $(r + s)(x)$ and $(r - s)(x)$ and evaluate $(r \cdot s)(1)$.

$(r + s)(x) =$

$(r - s)(x) =$

$(r \cdot s)(1) =$

Suppose the functions f and g are defined as follows.

$$f(x) = \frac{3}{x+1} \qquad g(x) = \frac{5}{x}$$

a) Find $\dfrac{f}{g}$.

b) Give its domain using an interval or union of intervals. Simplify your answers.

Suppose the functions f and g are defined as follows.

$$f(x) = \frac{x}{x-5} \qquad\qquad g(x) = -\frac{x+9}{x-5}$$

a) Find $\dfrac{f}{g}$.

b) Give its domain using an interval or union of intervals. Simplify your answers.

Suppose the functions f and g are defined as follows.

$f(x) = -3x + 1$ $\qquad$ $g(x) = \sqrt{x+5}$

a) Find $(f \cdot g)(x)$.

b) Give its domain using an interval or union of intervals.

c) Find $(f + g)(x)$.

d) Give its domain using an interval or union of intervals.

Suppose the functions f and g are defined as follows.

$$f(x) = \frac{1}{5x^2 + 4} \qquad g(x) = \sqrt{2x + 5}$$

a) Find $(f + g)(x)$.

b) Give its domain using an interval or union of intervals.

a) Find $(f \cdot g)(x)$.

d) Give its domain using an interval or union of intervals.

Objective 14: Composition of Functions

Given two functions f and g, the **composite function**, denoted by $f \circ g$, is defined by

$(f \circ g)(x) =$ ________________________.

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

To find the domain of $f \circ g$:

1) Determine the domain of the inside function (the first function to be applied). If there are any domain restrictions for the inside function, these are also domain restrictions for $f \circ g$.

2) Form the composite function and see if there are any **additional** restrictions.

The functions s and t are defined as follows.

$s(x) = -x - 2$ $\qquad\qquad$ $t(x) = 2x^2 - 2$

Find the value of $s(t(4))$.

Suppose the functions p and q are defined as follows.

$p(x) = x^2 + 5$ $\qquad\qquad$ $q(x) = \sqrt{x+7}$

Find the following:

a) $(p \circ q)(2)$

b) $(q \circ p)(2)$

Suppose the functions f and g are defined as follows.

$f(x) = \frac{5}{x}, x \neq 0$ $\qquad$ $g(x) = x^2 - 9$

Find the composition of $(f \circ f)(x)$.

Find the composition of $(g \circ g)(x)$.

Suppose the functions g and h are defined as follows.

$g(x) = 4x + 5$ $\qquad\qquad$ $h(x) = \sqrt{x+4}$

Find the composition of $(g \circ h)(x)$.

Find the domain of $(g \circ h)(x)$. Use interval notation.

Suppose the functions f and g are defined as follows.

$$f(x) = \frac{x}{x+1} \qquad\qquad g(x) = \frac{11}{x}$$

Find the composition of $(f \circ g)(x)$. Simplify your answer as much as possible.

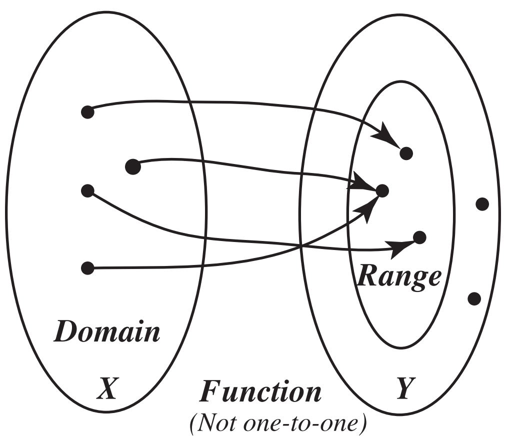
Find the domain of $(f \circ g)(x)$. Use interval notation.

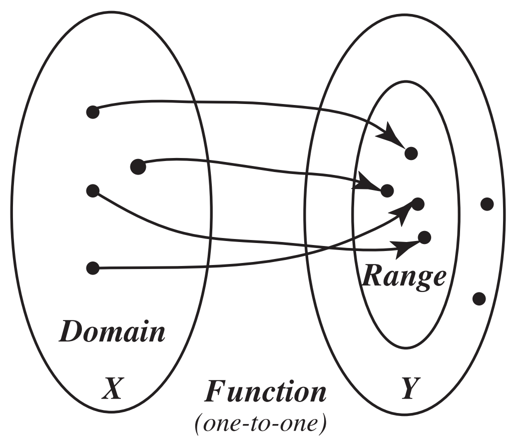
Objective 15: Inverse Functions

A function is **one-to-one** if any two different inputs in the domain correspond to two different outputs in the range.

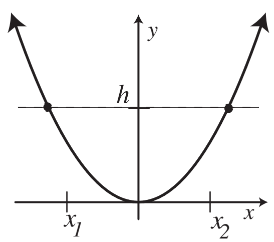
That is, if x_1 and x_2 are two different inputs of a function, then f is one-to-one if ______________.

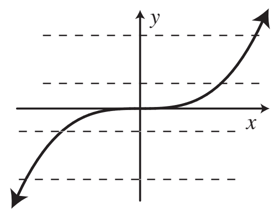
This can also be stated as, no y in the range is the image of more than one x in the domain.





For **functions** defined by an equation $y = f(x)$, the **Horizontal Line Test** can be used to determine if the graph represents the graph of a one-to-one function: If every horizontal line intersects the graph of a **function** in at most one point, then f is **one-to-one**.



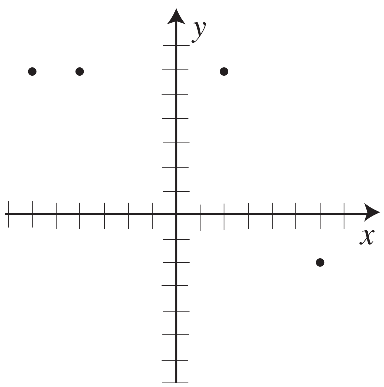


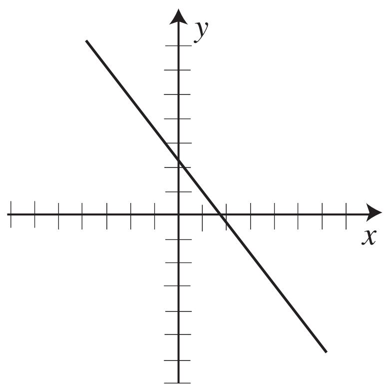
Determine whether each function is one-to-one.

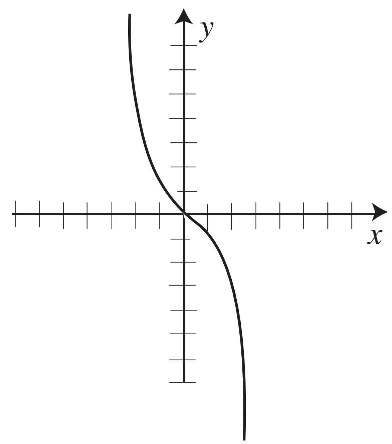
$\{(-2, 5), (-1, 3), (3, 3), (4, 12)\}$

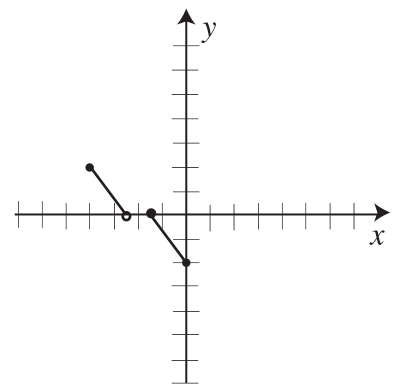
$\{(1, 2), (2, 8), (3, 18), (4, 32)\}$

For each function graphed below, state whether it is one-to-one.

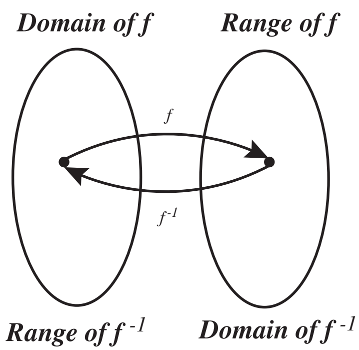








Definition of Inverse Functions Suppose that f is a one-to-one function. Then, corresponding to each x in the domain of f, there is exactly one y in the range (because f is a function); and corresponding to each y in the range of f, there is exactly one x in the domain (because f is one-to-one). The correspondence from the range of f back to the domain of f is called the **inverse function of f**. The symbol f^{-1} is used to denote the inverse function of f.



Based on the image, we can see:

the domain of f = range of f^{-1}

the range of f = domain of f^{-1}

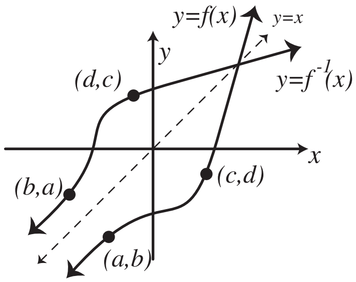
We can also see that starting with x, applying f, and then applying f^{-1} gets x back again.
Starting with x, applying f^{-1}, and then applying f gets x back again.

To put it simply, what f does, f^{-1} undoes, and vice versa.

In other words,

$(f^{-1} \circ f)(x) =$ ________________ $\quad (f \circ f^{-1})(x) =$ ________________

How are the graphs of f and f^{-1} related?



Two things to note:

If (a, b) is a point on the graph of $y = f(x)$, then the point ________________ is on the graph of $y = f^{-1}(x)$.

The graph of a one-to-one function f and the graph of its inverse function f^{-1} are symmetric with respect to the line ____________

Procedure for Finding the Inverse of a One-to-One Function

Step 1 Write y for $f(x)$ (to simplify the notation for the algebra steps to follow).

Step 2 Solve for x.

Step 3 Interchange x and y.

Step 4 Write f^{-1} for y.

The one-to-one functions g and h are defined as follows:

$g = \{(-9, 7), (-2, 8), (1, 3), (8, 9)\}$

$$h(x) = \frac{x - 3}{11}$$

Find the following:

a) $g^{-1}(8)$

b) $h^{-1}(x)$

c) $(g^{-1} \circ g)(1)$

Consider the function $f(x) = \sqrt{x+2} - 7$ for the domain $[-2, \infty)$.

a) Find $f^{-1}(x)$.

b) State the domain of $f^{-1}(x)$ in interval notation.

The one-to-one function f is defined below.

$f(x) = (x+6)^3$

Find $f^{-1}(x)$.

The one-to-one function f is defined below.

$f(x) = \sqrt[3]{x+1} + 10$

Find $f^{-1}(x)$.

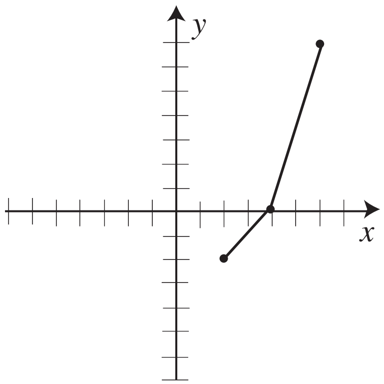
Consider the function $g(x) = \dfrac{3x - 4}{x + 7}$**.**

a) Find $g^{-1}(x)$.

b) State the domain of $g^{-1}(x)$ in interval notation.

Below is the entire graph of the function f.

Graph $f^{-1}(x)$.



Objective 16a: Exponential Functions

An **Exponential Function** is of the form $f(x) = a^x$, $a > 0$, $a \neq 1$

Note:

It is required that $a > 0$ (the base must be **positive**) otherwise the function would be ______________ for some x. i.e. ________________

It is required that $a \neq 1$ (the base cannot be 1) because for ________________ the graph would be linear, not exponential.

The exponential functions are classified into 2 groups, depending on the base.

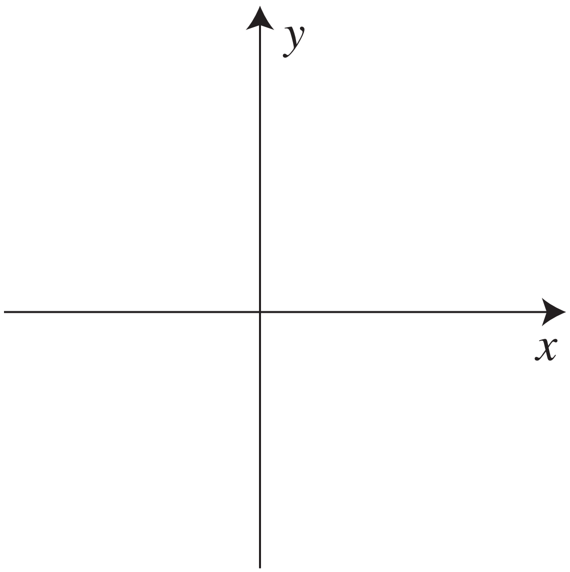
$f(x) = a^x$, $a > 1$

$f(x) = a^x$, $0 < a < 1$

We will consider two specific cases to illustrate the two families of functions.

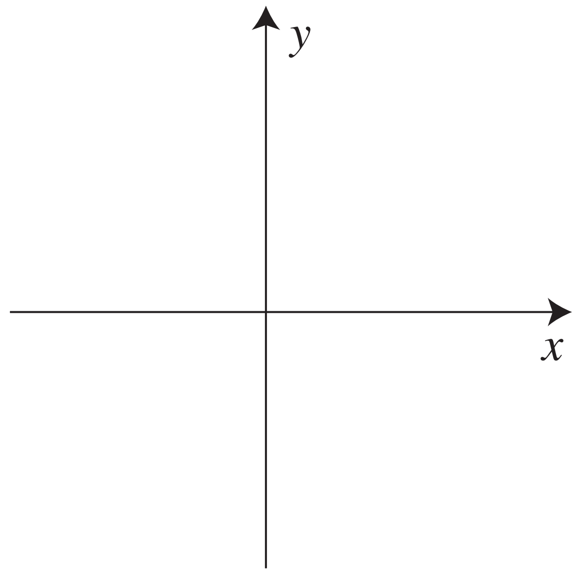
Consider $f(x) = 2^x$

for an example of $a > 1$



Consider $f(x) = \left(\frac{1}{2}\right)^x$

for an example of $0 < a < 1$



Properties of the Exponential Function $f(x) = a^x$, $a > 0$, $a \neq 1$

1. The domain is ____________ and the range is ____________.

2. There are ______ x-intercepts; the y-intercept is ______.

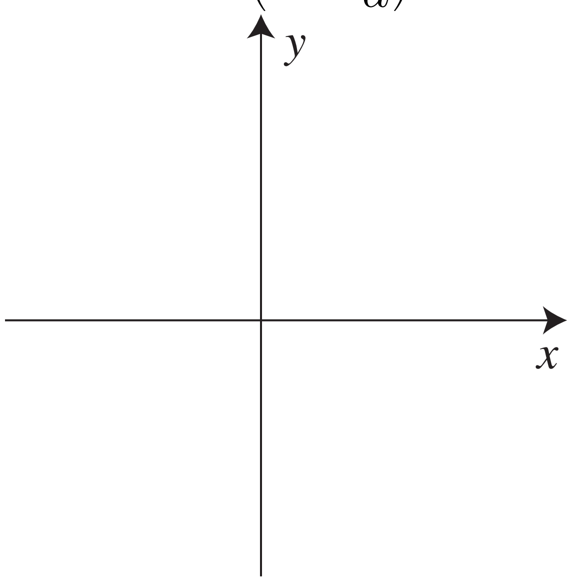
3. The x-axis (the horizontal line ______) is the horizontal asymptote.

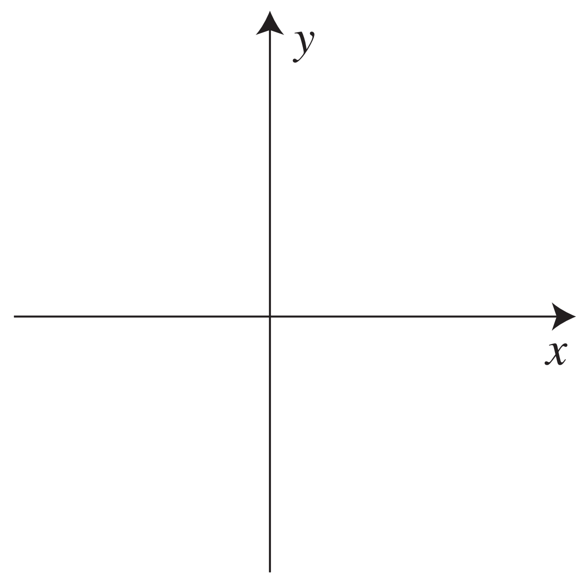
4. $f(x) = a^x$, $a > 1$, is an _______________ function and is one-to-one.

$f(x) = a^x$, $0 < a < 1$, is a _______________ function and is one-to-one.

5. The graph of f is smooth and continuous, with no corners or gaps.

6. The graph of every exponential function $f(x) = a^x$, $a > 0$, $a \neq 1$ passes through the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.



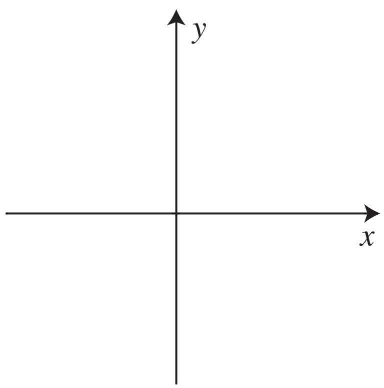


The irrational number e

The function $f(x) = e^x$ is often called "The" Exponential Function because of so many areas of application.

e is an irrational number, $e \approx 2.71828$, and is defined precisely as: $e = \lim_{n\to\infty} \left(1 + \dfrac{1}{n}\right)^n$

Graph $f(x) = e^x$



Graphing exponential functions with transformations or reflections:

Vertical Shifts (for $k > 0$):

$f(x) = a^x + k$

$f(x) = a^x - k$

Horizontal Asymptote shifts. (The range changes.)

Horizontal Shifts (for $h > 0$):

$f(x) = a^{x+h}$

$f(x) = a^{x-h}$

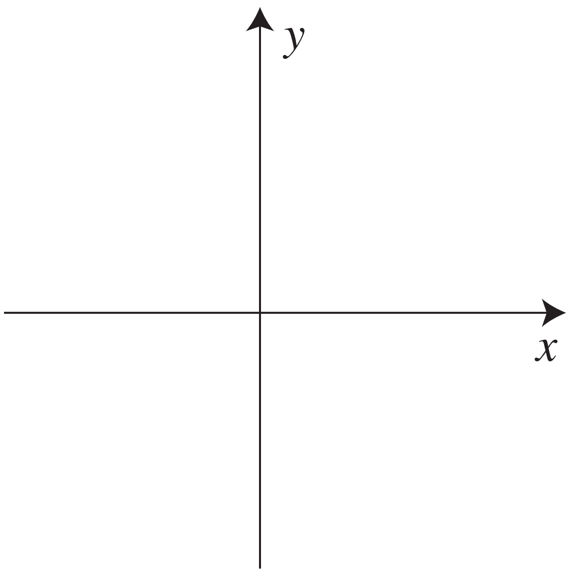
Horizontal Asymptote does not shift. (The range does not change)

Reflections:

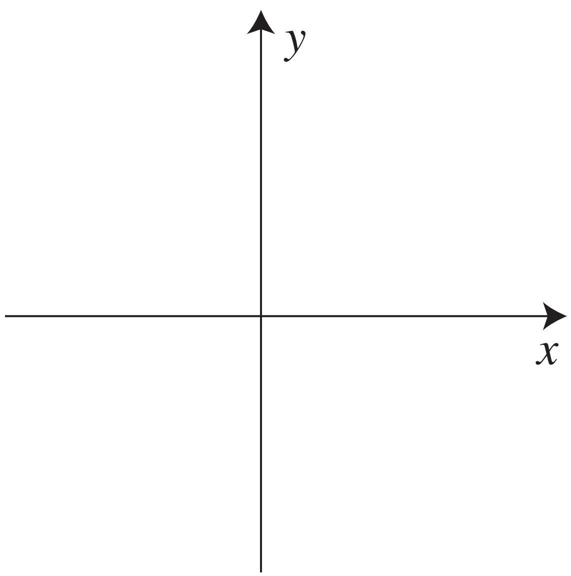
$f(x) = -a^x$

$f(x) = a^{-x}$

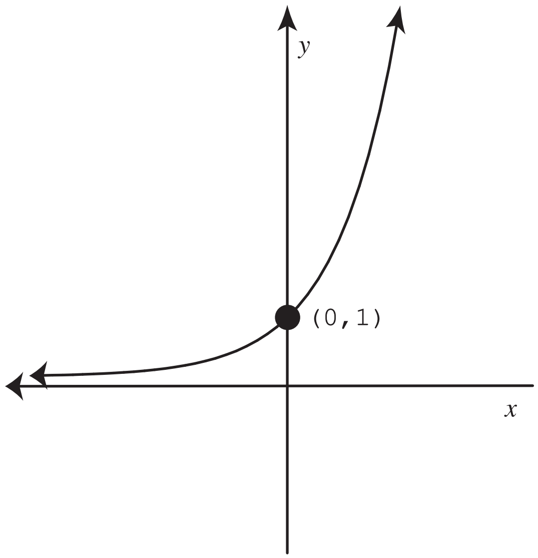
Graph the exponential function $f(x) = -\left(\frac{2}{3}\right)^x$. Plot five points on the graph of the function, and also draw the asymptote. Then click on the graph-a-function button.



Graph the exponential function $f(x) = 3^{-x}$. Plot five points on the graph of the function, and also draw the asymptote. Then click on the graph-a-function button.



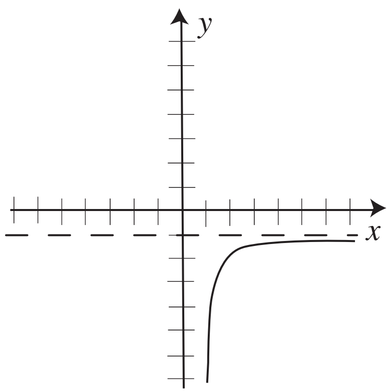
Given the graph of $y = 3^x$, translate it to become the graph of $y = 3^{x-1} - 2$.



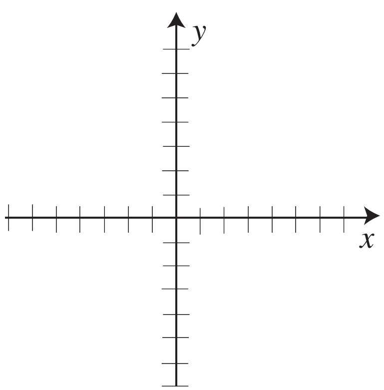
The graph of an exponential function is shown in the figure below.

The horizontal asymptote is shown as a dashed line.

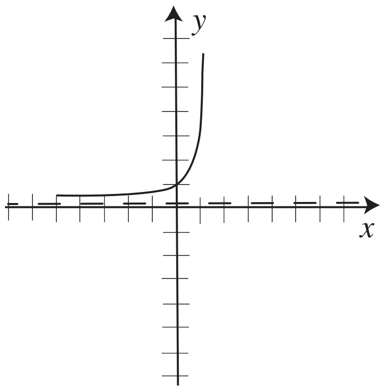
Find the range and the domain. Write your answers as inequalities, using x or y as appropriate.



Graph the function $g(x) = 4^{x-1}$ and give its domain and range using interval notation.

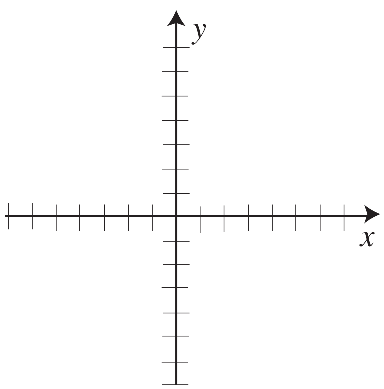


Below is the graph of $y = e^x$.



Transform it to make the graph of $y = e^{-x} - 6$.

Give the range and domain of $y = e^{-x} - 6$ using interval notation.



Objective 16b: Logarithmic Functions

Consider an exponential function $y = a^x$

What's the inverse function? Recall how to find the inverse. There is no algebraic operation to now solve for x.

We must define a new function. $y = \log_a x$ if and only if ______________.

Recall the relationships between the domain and range of inverse functions.

$$f(x) = a^x \qquad\qquad f^{-1}(x) = \log_a x$$

Domain:

Range:

Logarithms with base e or base 10

Common Logarithms are logarithms base 10; instead of ___________,

we write ___________.

Natural Logarithms are logarithms base e; instead of ___________, we write ___________.

Rewrite each exponential statement to an equivalent statement involving a logarithm.

$16 = 4^2$

$a^3 = 2.1$

$e^x = 8$

$10^a = b$

Rewrite each logarithmic statement to an equivalent statement involving an exponent.

$\log_3\left(\frac{1}{9}\right) = -2$

$\log_b 4 = 2$

$\ln 4 = x$

$\log x = 4$

Evaluate each expression.

$\log_8 8$

$\log_4 \left(\frac{1}{64}\right)$

$\log_6 36$

Find the exact value of each logarithm without using a calculator.

$\log \sqrt{1}0$ $\log_7 1$

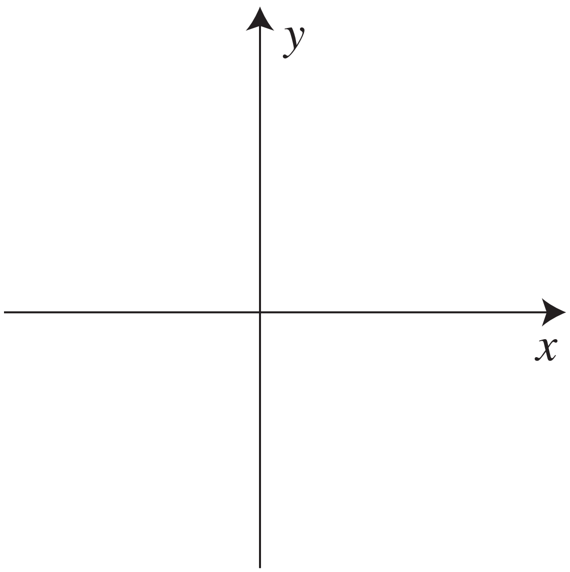
The logarithmic functions are classified into 2 groups, depending on the base.

$f(x) = \log_a x$, $a > 1$

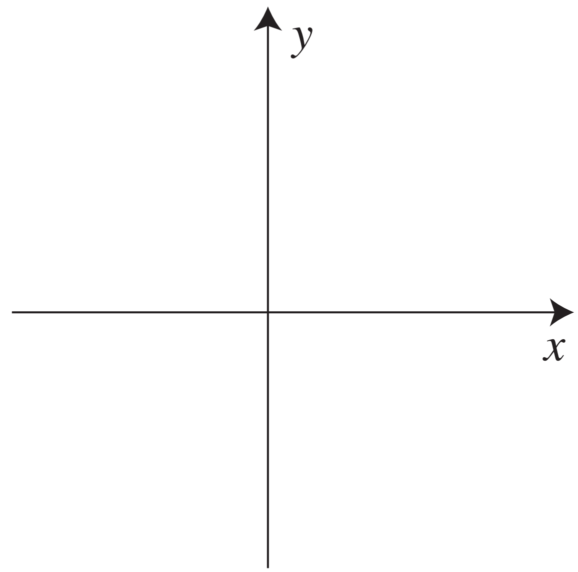
$f(x) = \log_a x$, $0 < a < 1$

We will consider two specific cases to illustrate the two families of functions.

Consider $f(x) = \log_2 x$
for an example of $a > 1$



Consider $f(x) = \log_{1/2} x$
for an example of $0 < a < 1$



Properties of the Logarithmic Function $f(x) = \log_a x$**,** $a > 0$**,** $a \neq 1$

1 The domain is ____________ and the range is ____________.

2 There are _______ y-intercepts; the x-intercept is _______.

3 The y-axis (the vertical line _______) is the vertical asymptote.

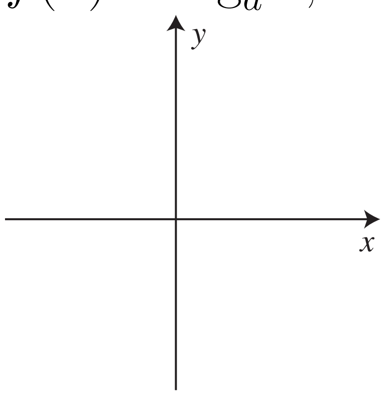
4 $f(x) = \log_a x$, $a > 1$, is an _____________ function and is one-to-one.

$f(x) = \log_a x$, $0 < a < 1$, is a _____________ function and is one-to-one.

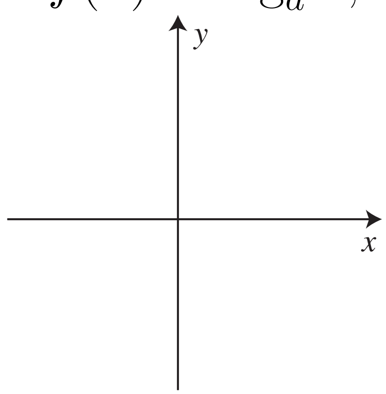
5 The graph of f is smooth and continuous, with no corners or gaps.

6. The graph of every logarithmic function $f(x) = \log_a x$, $a > 0$, $a \neq 1$ passes through the points $\left(\frac{1}{a}, -1\right)$, $(1, 0)$, and $(a, 1)$.

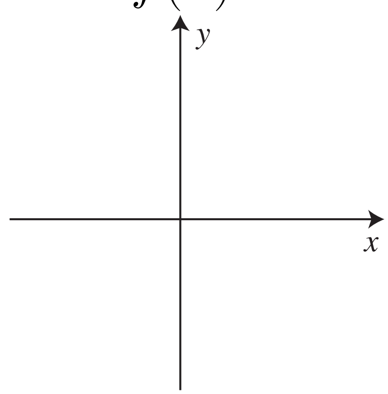
$f(x) = \log_a x$, $a > 1$



$f(x) = \log_a x$, $0 < a < 1$



$f(x) = \ln x$



Graphing logarithmic functions with transformations or reflections:

Vertical Shifts (for $k > 0$):

$$f(x) = \log_a x + k$$

$$f(x) = \log_a x - k$$

Vertical Asymptote does not shift. (The domain doesn't change.)

Horizontal Shifts (for $h > 0$):

$$f(x) = \log_a(x + h)$$

$$f(x) = \log_a(x - h)$$

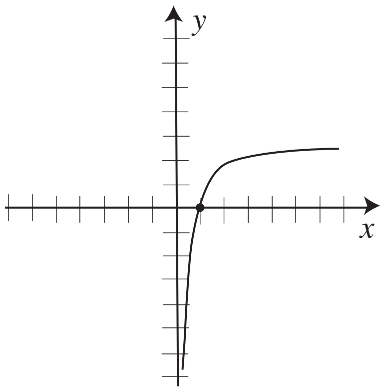
Vertical Asymptote shifts. (The domain changes.)

Reflections:

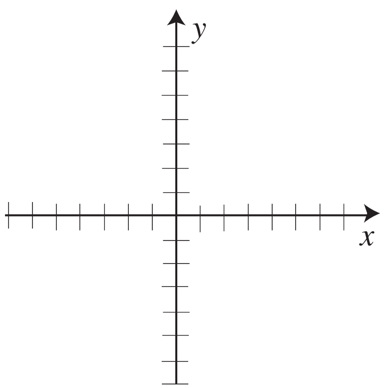
$$f(x) = -\log_a x$$

$$f(x) = \log_a(-x)$$

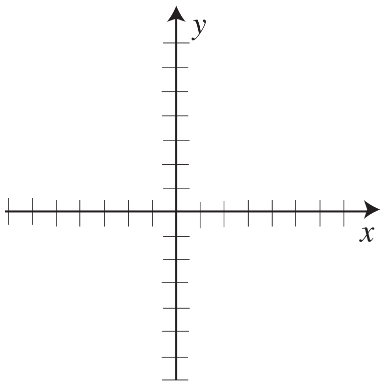
Below is the graph of $y = log_3\ x$.



Translate it to become the graph of $y = log_3\ (x - 4) + 3$.



Translate it to become the graph of $y = -1 + log_3\ (x + 3)$.



Domain of Logarithms:

To find the domain of a logarithmic function algebraically, set the argument of the logarithm __________ and solve for x.

Find the domain of each function. Write your answer as an interval or union of intervals.

$f(x) = \ln(x - 1)$ $\qquad$ $f(x) = log_4(8 - x)$ $\qquad$ $f(x) = \ln(-10 - x)$

Find the domain of each function.Write your answer as an interval or union of intervals.

$f(x) = \ln\sqrt{2x+5}$

$f(x) = \ln\left(\dfrac{9}{x-4}\right)$

Find the domain of each function.Write your answer as an interval or union of intervals.

$f(x) = log(x^2 - 16)$

$h(x) = \log_3 \left(\frac{x}{x-1} \right)$

Objective 16c: Properties of Logarithms

More Properties of Logarithms

As used below: $a > 0$, $a \neq 1$, $M > 0$, $N > 0$, $x > 0$, y and r represent any real number.

When Base and Argument Match:

$\log_a a =$ ________ $\log 10 =$ ________ $\ln e =$ ________

When Argument is 1:

$\log_a 1 =$ ________ $\log 1 =$ ________ $\ln 1 =$ ________

Inverse Function Properties:

$a^{\log_a M} =$ ________ $\log_a a^r =$ ________

$e^{\ln M} =$ ________ $\ln e^r =$ ________

Product Rule: $\log_a(MN) =$ ________________________

Must Note: $\log_a(MN) \neq$ ________________________

Must Note: $\log_a(M+N) \neq$ ________________________

Quotient Rule: $\log_a\left(\frac{M}{N}\right) =$ ________________________

Must Note: $\log_a\left(\frac{M}{N}\right) \neq$ ________________________

Must Note: $\log_a(M-N) \neq$ ________________________

Power Rule $\log_a M^r =$ ________________________

Use the properties of logarithms to expand the following:

Each logarithm should involve only one variable and should not have any radicals or exponents. Assume that all variables are positive.

$log(zy^5)$

$log\left(\frac{x}{z^5}\right)$

Each logarithm should involve only one variable and should not have any radicals or exponents. Assume that all variables are positive.

$$\ln\left(\frac{y^7}{\sqrt{xz^3}}\right) \qquad\qquad log\sqrt{\frac{(x+4)^5}{x^3}}$$

Each logarithm should involve only one variable and should not have any radicals or exponents. Assume that all variables are positive.

$log\sqrt[3]{x^5y^2z}$

Write each expression as a single logarithm.

$$3\ log_4\ x - \frac{1}{5}\ log_4\ w + 7\ log_4\ z$$

$$3\ log_m\ x + 2\ (log_m\ y - 4\ log_m\ z)$$

Write each expression as a single logarithm.

$$7\ log_b\ (7z+1) + \frac{1}{4}\ log_b\ (x+7)$$

Objective 16d: Applications of Exponential and Logarithmic Functions

---

If the rate of inflation is 2.5% per year, the future price $p(t)$ (in dollars) of a certain item can be modeled by the following exponential function, where t is the number of years from today.

$p(t) = 1500(1.25)^t$

Find the current price of the item and the price 9 years from today. Round your answers to the nearest dollar as necessary.

Suppose that \$5000 is placed in a savings account at an annual rate of 8.6%, compounded semiannually. Assuming that no withdrawals are made, how long will it take for the account to grow to \$7945?

Do not round any intermediate computations, and round your answer to the nearest hundredth.

There was sample of 650 milligrams of a radioactive substance to start a study. Since then, the sample has decayed by 1.3% each year. Let t be the number of years since the start of the study. Let y be the mass of the sample in milligrams.

Write an exponential function showing the relationship between y and t.

The number of milligrams $D(h)$ of a certain drug that is in a patient's bloodstream h hours after the drug is injected is given by the following function.

$$D(h) = 50e^{-0.15h}$$

When the number of milligrams reaches 33, the drug is to be injected again. How much time is needed between injections? Round your answer to the nearest tenth, and do not round any intermediate computations.

Objective 16e: Logarithmic Equations

Solving Logarithmic Equations.

There are two different types of logarithmic equations that we are going to work with:

1) Logarithmic equations that contain all log terms.

2) Logarithmic equations that do not contain all log terms.

It is important to note: Logarithms cannot have arguments which are negative or zero, but quadratics and other equations can have negative solutions. When we convert a log equation to a different type of equation by equating the insides of the logs, we may be "creating" solutions that didn't previously exist. Because of this, we need to check the solutions for logarithmic equations.

Solving logarithmic equations that contains all log terms. *Remember to check your solutions.

Logarithmic functions are one-to-one; that means:

____________________ if and only if ____________________

Solve for x.

$log(x+2) - log\ 3 = log\ x$

$\ln 16 = \ln(x-3) - \ln 19$

Solve for x.

$$log_7\ 14 = log_7\ 4 + log_7\ (x+3)$$

Solve for x.

$$log_4\ (x-1) - log_4\ 2 = log_4\ x$$

Solving logarithmic equations that do NOT contain all log terms. *Remember to check your solutions.

When a logarithmic equation does not contain all log terms, we can use the definition of logarithms to covert it to an exponential equation.

Solve for x.

$log_3\ (4x - 19) = 4$ $\qquad\qquad$ $-2log_3\ (4x) = -6$

Solve for x.

$3\ log_2\ (x + 18) = 12$

$-4 + log_3\ (x + 1) = -3$

Solve for x.

$$log_2\ (x-5) = 3 - log_2\ (x-7)$$

Objective 16f: Exponential Equations

Solving Exponential Equations

There are two different equations that we are going to work with:

1) Exponential equations where we can obtain the same base.

2) Exponential equations where we can not obtain the same base.

Solving exponential equations where we can obtain the same base.

Exponential functions are one-to-one; that means:

____________________ if and only if ____________________

Rewrite each side (if needed) in terms of a common base. Be sure to rename as equals.

Solve for x.

$3^{-x} = 81$

$64 = 32^{-x+2}$

$4^{-9x} = 64^{2-4x}$

Solve for x.

$4^{x^2-11x-6} = 64^{5-5x}$

$8^{4-9x} = 2^{x^2-38x+42}$

Solving exponential equations where we can NOT obtain the same base.

We are going to use logarithms and natural logarithms to solve equations where we can not obtain the same base.

Solve for x. Round your answer to the nearest hundredth. Do not round any intermediate computations.

$e^{x+9} = 5$ $\quad$ $e^{9x} = 9$ $\quad$ $e^{-9y} = 12$

Solve for x. Write the exact answer using either base-10 or base-e logarithms.

$3^{-x-6} = 5^{-8x}$

$14^{10x} = 15^{-x-8}$

Solve for x. Write the exact answer using either base-10 or base-e logarithms.

$5^{x-9} = 13^{4x}$

Objective 17a: Systems of Equations

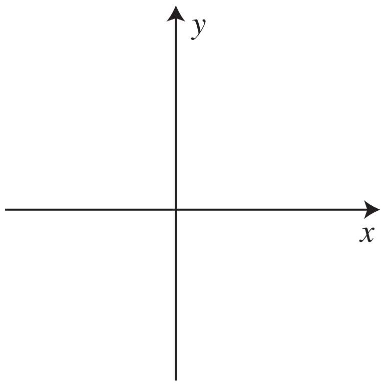
A **solution** to a system of linear equations consists of values for the variable that are solutions of **each** equation.

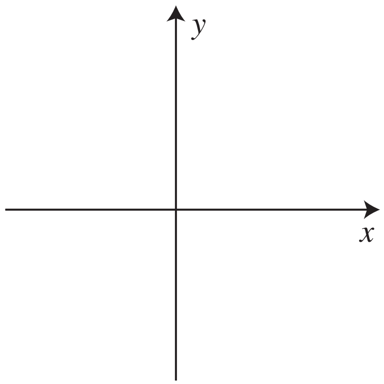
When solving two linear equations in two unknowns, there are 3 possible types of solution.

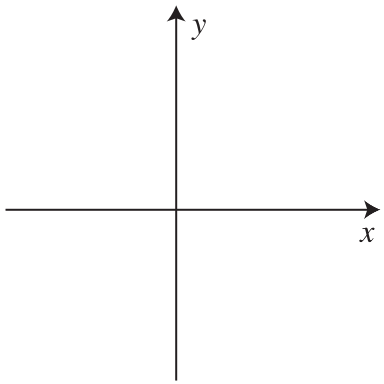
When a system of linear equations has **one** solution, it is said to be ______________.

When a system of linear equations has **no** solution, it is said to be ______________.

When a system of linear equations has **infinite** solutions, it is said to be ______________.







To solve algebraically we will use the ________________________________

Multiply one, or both equation if needed, by non-zero numbers so that when the equations are added, one variable is eliminated.
That is, get the coefficients of one of the variables to be negatives of each other.

For each ordered pair, determine whether it is a solution to the system of equations.

$$\begin{cases} 3x - 2y = -9 \\ 2x + 5y = -6 \end{cases}$$

$(7, -4)$ $(-3, 0)$ $(6, 2)$ $(-5, -3)$

Solve the following system of equations.

$$\begin{cases} 3x - 8y = 3 \\ -3x + 13y = -18 \end{cases}$$

$$\begin{cases} 2x - 5y = 1 \\ 7x + 6y = -20 \end{cases}$$

Solve each system of equations.

$$\begin{cases} 4x - y = 5 \\ 3x + 2y = 12 \end{cases}$$

$$\begin{cases} \dfrac{1}{3}x + \dfrac{1}{2}y = 7 \\ \dfrac{1}{5}x - 2y = -\dfrac{2}{5} \end{cases}$$

Objective 17b: Systems of Inequalities

The **graph of an inequality in two variables** x and y consists of all points (x, y) whose coordinates satisfy the inequality.

Steps for Graphing an Inequality

Step 1 Replace the inequality symbol by an equal sign, and graph the resulting equation. If the inequality is strict ($<$ or $>$) use ________________; if it is nonstrict ($\leq$ or $\geq$) use a ________________ mark. This graph separates the xy-plane into two or more regions.

For a linear inequality, the line separates the xy-plane into two regions called

____

Step 2 In each region, select a test point, P.

If the coordinates of P satisfy the inequality, so do all the points in that region. Indicate this by shading the region.

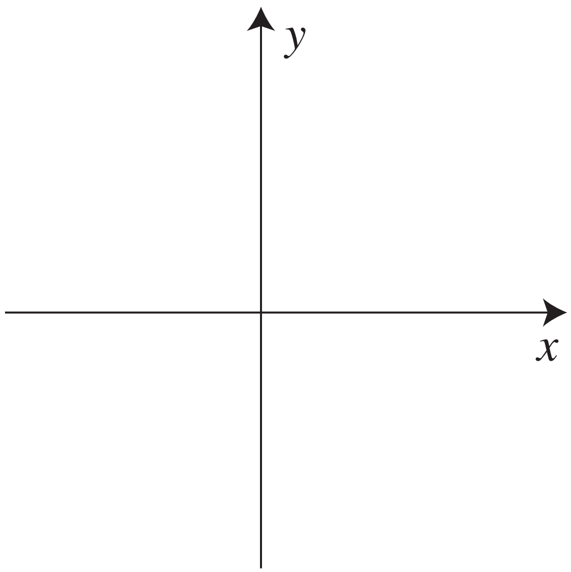
If the coordinates of P do not satisfy the inequality, none of the points in that region does.

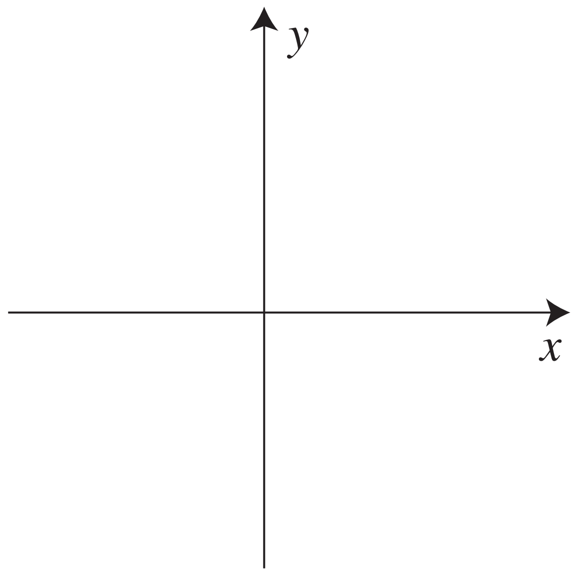
Graph each inequality.

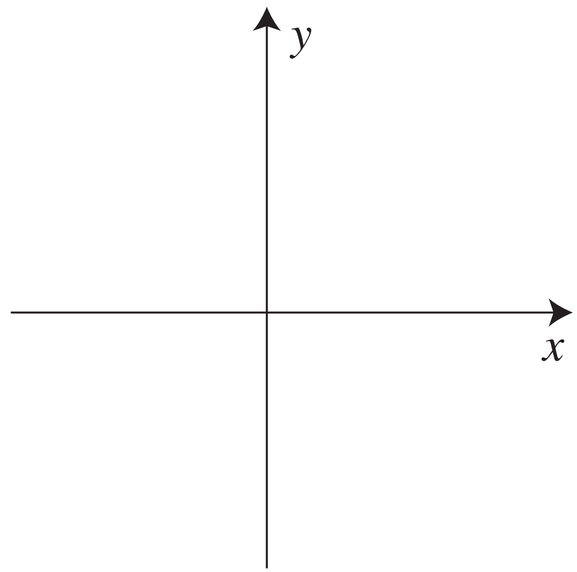
$y < 7$

$y \leq -5x + 2$

$2x + 5y > 10$





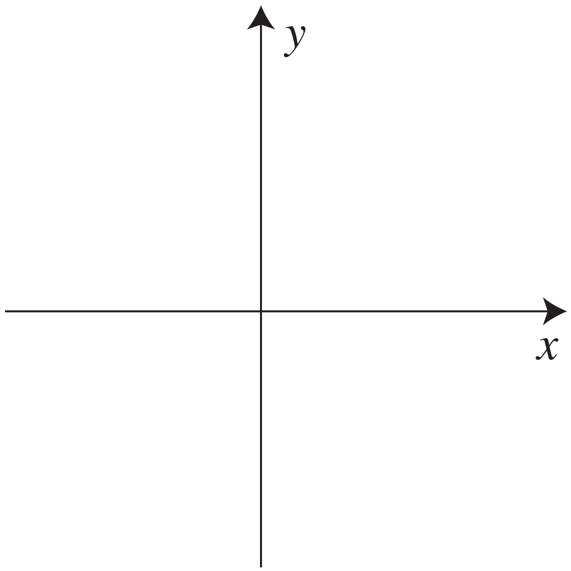


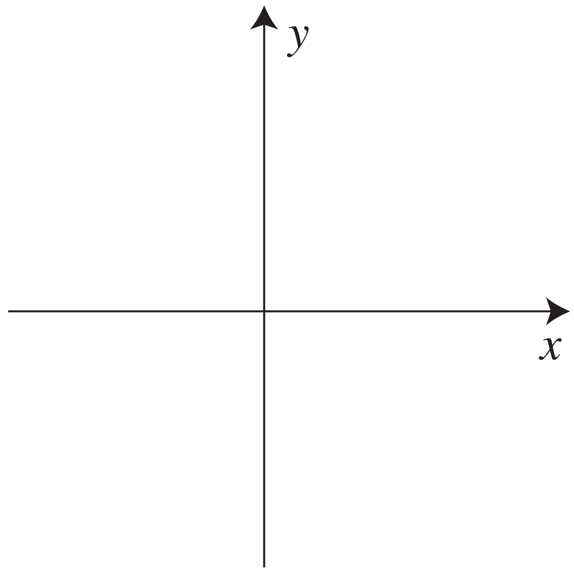
Graph each inequality.

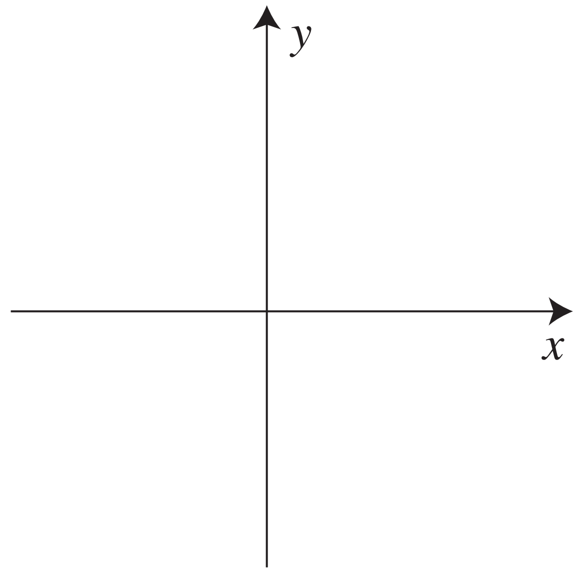
$y > -x^2+5$

$y < -3x^2-12x-7$

$(x+1)^2+y^2 < 9$

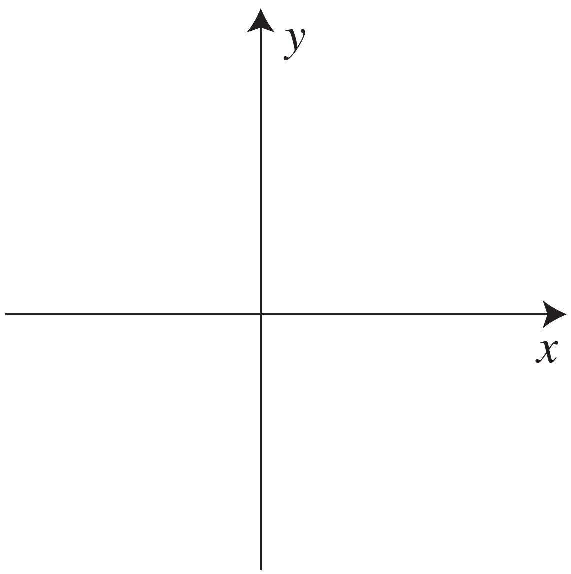






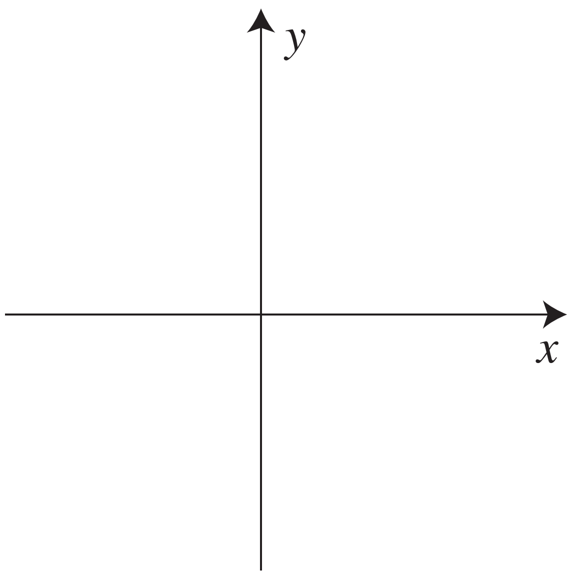
Graph the system of inequalities.

$$\begin{cases} y < 4x + 3 \\ y \geq -3x + 7 \end{cases}$$



Graph the system of inequalities.

$$\begin{cases} 4x + 5y > 5 \\ -7x + 3y \geq -6 \end{cases}$$



Graph the system of inequalities.

$$\begin{cases} x + y > -1 \\ x^2 + y^2 \leq 1 \end{cases}$$