1.1.16 Show that there are no retractions \( r : X \to A \) in the following cases:

@ \( X = \mathbb{R}^3 \) with \( A \) any subspace homeomorphic to \( S^1 \).

Suppose \( \exists r: X \to A \)

\[
\begin{array}{c}
A \\ i \\ r \\
\downarrow \\
\uparrow
\end{array}
\]

nontrivial \( r \circ i = 1_A \)

\[
\begin{array}{c}
\pi_1(A, x_0) \\ i^* \\ r^* \\
\downarrow \\
\uparrow
\end{array}
\]

trivial

\[
\begin{array}{c}
\pi_1(X, x_0) \\ r^* \\
\downarrow \\
\uparrow
\end{array}
\]

nontrivial

\[
(r \circ i)_* = r_* \circ i_*
\]

\[
(1_A)_* = 1_A
\]

\( X = \mathbb{R}^3 \) is simply connected since it is path-connected and every loop is path homotopic to the constant loop by the straight-line homotopy.

\( \Rightarrow \pi_1(X, x_0) = 1 \) is trivial

Since \( \pi_1(A, x_0) \) is nontrivial we have a contradiction.