Errata for Shocks and Rarefactions Arise in a Two-Phase Model with Logistic Growth

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Abstract
This errata is necessary to address a crucial typo and to discuss a minor error. In equation (5), there is a missing derivative which can make reproduction of these results difficult to attain. Next, our particular choice of \( \Gamma(t) = 0 \) to produce this result in this paper are physically irrelevant. Instead, we make a choice of \( \Gamma(t) = 1 \), in which we see, similar results can be achieved as those produced in the paper.

1. The Two-Phase Model

This is the reduced two-phase model given in the original paper.

\[
\frac{\partial}{\partial t}(\phi_1) + \frac{\partial}{\partial x}(u\phi_1) = G(\phi_1, 1 - \phi_1), \quad (1)
\]

\[
-\frac{\partial}{\partial t}(\phi_1) + \frac{\partial}{\partial x}(v(1 - \phi_1)) = -G(\phi_1, 1 - \phi_1), \quad (2)
\]

\[
\mu_1(1 - \phi_1) \frac{\partial}{\partial x} \left( \phi_1 \frac{\partial}{\partial x} u \right) - \mu_2 \phi_1 \frac{\partial}{\partial x} \left( (1 - \phi) \frac{\partial}{\partial x} v \right) = Q \quad (3)
\]

where

\[
Q = \xi \phi_1(1 - \phi_1)(u - v) + k_2 \phi \frac{\partial}{\partial x} \phi_1(1 - \phi)(3\phi_1 - 2\phi_0)
\]

The osmotic pressure function (not given explicitly in the original paper) takes the form \( \psi(\phi_1) = k_2 \phi_1^2(\phi_1 - \phi_0) \), where \( \phi_0 \) is a reference volume fraction.

2. General Reduction

The transformation we found is

\[ u(t, x) = \frac{\Gamma(t)}{\alpha} + f \left( x - \frac{1}{\alpha} \int \Gamma(t) dt \right), \]
\[ v(t, x) = \frac{\Gamma(t)}{\alpha} + g \left( x - \frac{1}{\alpha} \int \Gamma(t) dt \right), \]
\[ \phi_1(t, x) = m \left( x - \frac{1}{\alpha} \int \Gamma(t) dt \right), \]

where \( \Gamma \), assumed to be smooth, is an arbitrary function of \( t \), and \( \alpha \) is an arbitrary constant. Applying the transformation given by (4) to (1-3) reduces the system to the ordinary differential equations given by

\[ (mf)' - G = 0, \]
\[ ((1 - m)g)' + G = 0, \]
\[ \mu_1(1 - m)(mf)' - \mu_2m((1 - m)g)' \]
\[ -k_2(1 - m)m'(3m - 2\phi_0) - \xi m(1 - m)(f - g) = 0, \]

where \( m, f, \) and \( g \) are all functions of \( r = x - \frac{1}{\alpha} \int \Gamma(t) dt \) that are to be determined.

3. Logistic Growth in an Inviscid System

Under this section we made the following case for \( \Gamma(t) = 0 \), but this particular choice trivializes the invariant surface condition. In other words, to derive the transformation (4), we need to solve the system

\[ \alpha u_t + \Gamma(t)u_x = \frac{d}{dt} \Gamma(t), \]
\[ \alpha v_t + \Gamma(t)v_x = \frac{d}{dt} \Gamma(t), \]
\[ \alpha \phi_{1t} + \Gamma(t)\phi_{1x} = 0. \]

Instead, we present the case of \( \Gamma(t) = 1 \), and let \( \alpha = 1000 \). With these choices, the graphs remain virtually unchanged.
3.1. $\Gamma(t) = 1$

For

$$u(\phi_1(x, 0), 0) = \begin{cases} 
0 & \text{if } x < 0 \\
x & \text{if } 0 < x < .5 \\
0.5 & \text{if } x > .5, 
\end{cases}$$  \quad (10)$$

we have

Figure 1: For initial conditions given by (10) (top), this shows the characteristic curves for growth given by $k_1 = 1$ (left) and $k_1 = 5$ (right), producing shocks. As growth increases, we see more rapid shockwaves with high frequencies, where as small growth is slow to produce shocks and have lower frequencies.
And with
\[
u(\phi(x,0), 0) = \begin{cases} 
0.5 & \text{if } x < 0 \\
x & \text{if } 0 < x < .5 \\
0 & \text{if } x > .5, 
\end{cases}
\] (11)
we get

Figure 2: For initial conditions given by (11), the characteristic curves for growth given by \(k_1 = 1\) (left) and \(k_1 = 5\) (right) we see rarefactions. As growth increases, we see the loss of information between characteristics increases with wider rarefactions.