Errata for Shocks and Rarefactions Arise in a Two-Phase Model with Logistic Growth

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Abstract

This errata is necessary to address a crucial typo and to discuss a minor error. In equation (5), there is a missing derivative which can make reproduction of these results difficult to attain. Next, our particular choice of $\Gamma(t) = 0$ to produce this results in this paper are physically irrelevant. Instead, we make a choice of $\Gamma(t) = 1$, in which we see, similar results can be achieved as those produced in the paper.

1. The Two-Phase Model

This is the reduced two-phase model given in the original paper.

$$\frac{\partial}{\partial t}(\phi_1) + \frac{\partial}{\partial x}(u\phi_1) = G(\phi_1, 1-\phi_1), \quad (1)$$

$$-\frac{\partial}{\partial t}(\phi_1) + \frac{\partial}{\partial x}(v(1-\phi_1)) = -G(\phi_1, 1-\phi_1), \quad (2)$$

$$\mu_1(1-\phi_1)\frac{\partial}{\partial x}\left(\phi_1\frac{\partial}{\partial x}u\right) - \mu_2\phi_1\frac{\partial}{\partial x}\left((1-\phi)\frac{\partial}{\partial x}v\right) = Q \quad (3)$$

where

$$Q = \xi\phi_1(1-\phi_1)(u-v) + k_2\phi\frac{\partial}{\partial x}\phi_1(1-\phi)(3\phi_1 - 2\phi_0)$$

The osmotic pressure function (not given explicitly in the original paper) takes the form $\psi(\phi_1) = k_2\phi_1^2(\phi_1 - \phi_0)$, where ϕ_0 is a reference volume fraction.

2. General Reduction

The transformation we found is

$$\begin{aligned} u(t,x) &= \frac{\Gamma(t)}{\alpha} + f\left(x - \frac{1}{\alpha}\int \Gamma(t)dt\right), \\ v(t,x) &= \frac{\Gamma(t)}{\alpha} + g\left(x - \frac{1}{\alpha}\int \Gamma(t)dt\right), \\ \phi_1(t,x) &= m\left(x - \frac{1}{\alpha}\int \Gamma(t)dt\right), \end{aligned} \tag{4}$$

where Γ, assumed to be smooth, is an arbitrary function of t, and α is an arbitrary constant. Applying the transformation given by (4) to (1-3) reduces the system to the ordinary differential equations given by

$$\begin{aligned} &(mf)' - G = 0, \\ &((1-m)g)' + G = 0, \qquad (5) \\ &\mu_1(1-m)\left(mf'\right)' - \mu_2 m\left((1-m)g'\right)' \\ &-k_2(1-m)m'(3m - 2\phi_0) - \xi m(1-m)(f-g) = 0, \qquad (6) \end{aligned}$$

where m, f, and g are all functions of $r = x - \frac{1}{\alpha}\int \Gamma(t)dt$ that are to be determined.

3. Logistic Growth in an Inviscid System

Under this section we made the following case for $\Gamma(t) = 0$, but this particular choice trivializes the invariant surface condition. In other words, to derive the transformation (4), we need to solve the system

$$\alpha u_t + \Gamma(t)u_x = \tfrac{d}{dt}\Gamma(t), \tag{7}$$

$$\alpha v_t + \Gamma(t)v_x = \tfrac{d}{dt}\Gamma(t), \tag{8}$$

$$\alpha \phi_{1_t} + \Gamma(t)\phi_{1_x} = 0. \tag{9}$$

Instead, we present the case of $\Gamma(t) = 1$, and let $\alpha = 1000$. With these choices, the graphs remain virtually unchanged.

3.1. $\Gamma(t) = 1$

For

$$u(\phi_1(x,0),0) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 < x < .5 \\ 0.5 & \text{if } x > .5, \end{cases} \tag{10}$$

we have

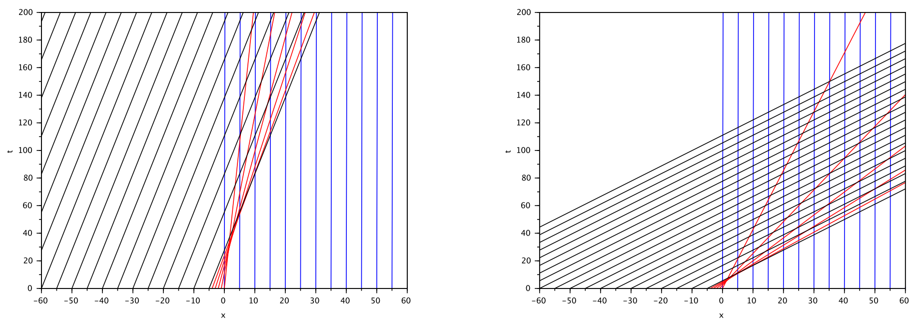

Figure 1: For initial conditions given by (10) (top), this shows the characteristic curves for growth given by $k_1 = 1$ (left) and $k_1 = 5$ (right), producing shocks. As growth increases, we see more rapid shockwaves with high frequencies, where as small growth is slow to produce shocks and have lower frequencies.

And with

$$u(\phi(x,0),0) = \begin{cases} 0.5 & \text{if } x < 0 \\ x & \text{if } 0 < x < .5 \\ 0 & \text{if } x > .5, \end{cases} \tag{11}$$

we get

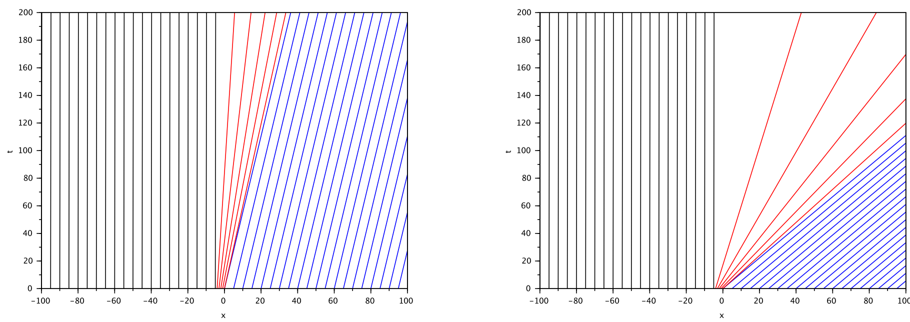

Figure 2: For initial conditions given by (11), the characteristic curves for growth given by $k_1 = 1$ (left) and $k_1 = 5$ (right) we see rarefactions. As growth increases, we see the loss of information between characteristics increases with wider rarefactions.