# Midterm I

MAP 4341 Fall 2008

Full Name:________________________ Score: __________

**Show all of your work for full credit.**

1. Determine the value of $\beta$ for which the following PDE has a steady-state solution:

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \frac{\partial}{\partial x}\left(\frac{1}{x+1}\frac{\partial u}{\partial x}\right) - 1 \\
u(0,t) &= 0 \\
\frac{\partial u}{\partial x}(L,t) &= \beta
\end{aligned}$$

2. Determine the equilibrium temperature distribution inside a circular annulus ($r_1 \leq r_2$) where the temperature at the outer radius is kept at temperature $T$ and the inner radius is perfectly insulated.

3. Determine, but do not solve, the ordinary differential equations found applying the method of separation of variables to the following PDE's:

   (a)

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - v_0\frac{\partial u}{\partial x}$$

   (b)

$$\frac{\partial^2 u}{\partial t^2} = k\frac{\partial^2 u}{\partial x^2}$$

4. Find the (real) eigenvalues and corresponding eigenfunctions of the problem

$$\begin{aligned} \frac{d^2\phi}{dx^2} &= \lambda \\ \frac{d\phi}{dx}(0) + \phi(0) &= 0 \\ \phi(L) &= 0 \end{aligned}$$

5. Complete the following parts to solve Laplace's equation on the inside of the 60° wedge of radius $a$, subject to the boundary conditions:

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\pi/3) &= 0 \\ u(a,\theta) &= f(\theta) \end{aligned}$$

(a) Derive the ordinary differential equations implied by separation of variables (including the boundary conditions).

(b) Show that 0 is NOT an eigenvalue of the eigenvalue problem.

You may now assume that the eigenvalues/eigenfunctions of the implied eigenvalue problem are: $9n^2$ and $\sin(3n\theta)$ (respectively), $n = 1, 2, 3...$ and that the general solution to the other ODE is $G(r) = cr^n$.

(c) Use superposition to write the solution to Laplace's equation and determine the coefficients that are required in order to satisfy the inhomogeneous boundary condition.