Midterm 1

M224 Spring 2004

Full Name:____________________________ Score: ____________

Show all of your work for full credit.

1. Solve the initial value problem

$$\begin{aligned} \frac{dy}{dt} &= ty^2 \\ y(0) &= 1 \end{aligned}$$

2. Solve the differential equation

$$y' = \cos(x) - y\cos(x)$$

3. Given the ODE $x' = x + e^{-t}$,

   (a) Find the solution using the method of integrating factors.

   (b) Find the solution using variation of parameters.

4. Consider the equation $(x+y)dx - xdy = 0$.

(a) Show that the equation is NOT exact.

(b) Show that the equation resulting from multiplying the above equation by the function $\frac{1}{x^2}$ IS exact.

5. Show that solutions to the initial value problem

$$\begin{aligned} y' &= (y-1)(\cos(2y-t)) \\ y(0) &= 2 \end{aligned}$$

are NEVER zero.

Consider the ODE

$$y' = y(y-1)(y+1)$$

(a) What are the equilibrium solutions?

(b) Use the graph of the right-hand-side to draw the phase-line.

(c) What is the stability of each equilibrium?

(d) Sketch the qualitative behavior of the solution for several different initial conditions.

6. (a) Suppose a population of bacteria is growing exponentially at a rate k. Experiments have shown that the population doubles in T days. Find the rate as a function of T.

(b) If a biologist starts with 20,000 bacteria growing at a rate $k = .7$ bacteria/day and removes bacteria at a constant rate of R bacteria per hour. If the biologists wants to be certain that there will be bacteria in the culture after 10 days, what is the maximum rate of removal R?