Midterm II

Math 224 - Section 05 — Spring 2003

Full Name:______________________________ Score: ____________

Show all of your work for full credit.

1. Show that $y_1(t) = \cos(2t)$ and $y_2(t) = \sin(2t)$ form a fundamental set of solutions for the ODE

$$y'' + 4y = 0,$$

then find a solution satisfying $y(0) = 1$ and $y'(0) = 0$.

2. Solve the following ODE

$$y'' + 4y' + 5y = 0$$

3. Solve the following ODE

$$y'' + 5y' + 6y = 0$$

4. Consider the ODE

$$y'' + 4y' + 4y = 4 - t$$

(a) Solve the homogeneous equation.

(b) Find a particular solution.

(c) Find the general solution to the ODE.

Describe the steps that required to solve the ODE

$$y'' + ay' + by \quad = \quad f(t)$$

using variation of parameters.

5. Use the definition of the Laplace transform to find the Laplace transform of the following functions

(a) $f(t) = e^{-at}$ for constant a.

(b) $g(t) = 3t$

6. Prove that

$$\mathcal{L}tf(t)(s) = -F'(s)$$

For f piecewise continuous function of exponential order with Laplace transform $F(s)$. (Hint: Start with the right-hand-side)

Use this fact to calculate the Laplace transform of $g(t) = t(e^{-at} + 3t)$.

7. Find the inverse Laplace transform of $G(s) = \frac{1}{s^2 - 2s - 3}$, $s > 3$.

8. Solve the following ODE using Laplace transforms

$$\begin{aligned} y' + y &= te^t \\ y(0) &= -2 \end{aligned}$$