Midterm 1

Math 224 - Section 05 Spring 2003

Full Name:______________________________ Score: ______________

Show all of your work for full credit.

1. Find the general solution to the differential equation

$$y' = \frac{y}{x-1} + \frac{y}{x+1}$$

2. Find the general solution to the differential equation

$$x' = x \sin t + 2te^{-\cos t}$$

3. Find the particular solution to the differential equation

$$\begin{aligned} x' - \frac{n}{t}x &= e^t t^n \ , \text{ n a positive integer} \\ x(1) &= 1 \end{aligned}$$

4. Given the differential equation

$$y\ dx + (2x - ye^y)\ dy \ = \ 0$$

(a) Show that $\mu = y$ is an integrating factor.

(b) Solve the resulting exact differential equation. You will probably need to know that $\int y^2 e^y dy = y^2 e^y - 2ye^y + 2e^y + C$.

Choose one of the next two problems. Indicate your choice by marking yes or no in the space provided.

5. grade(yes/no): ____________

A young person opens an account with 1000$. The account earns $r\%$ interest compounded continuously.

(a) Find the time required for the initial deposit to double as a function of r assuming that there are no deposits.

(b) Suppose that the same person also deposits k dollars per year into the account. Determine k so that one million dollars will be available for retirement in 40 years given that $r = 7.5\%$.

Choose either this problem or the previous problem. Indicate your choice by marking yes or no in the space provided.

6. grade(yes/no): ____________

In the investigation of a homicide it is often important to estimate the time of death. One method used is to approximate the rate of change of the body temperature by Newton's law of cooling. That is

$$\frac{d\theta}{dt} = -k(\theta - T),$$

where $\theta(t)$ is the body temperature at time t, T is the ambient temperature and $k > 0$ is a constant.

(a) Solve the differential equation (Note:The differential equation is linear, so there are at least two methods one can use).

(b) Using the solution found in the previous part, find the particular solution given that the temperature of the corpse is $85°F$ when discovered. Assume that the ambient temperature,T, is $68°$.

(part c is on the next page)

(c) Using the previous parts, and that $k = .52$, approximately how long ago was the corpse murdered, assuming that the temperature of the living person was $98.6°F$?

7. Given the differential equation

$$\frac{dN}{dt} = -r(1-\frac{N}{K})(1-\frac{N}{K+1})N$$

where r and K are positive constants.

(a) Use the graph of the right-hand-side to draw the phase line.

(b) Use the previous part to sketch the qualitative behavior of solutions, for different initial conditions.

(c) Classify the equilibrium solutions.

Bonus: Find the implicit solution of

$$\frac{dN}{dt} = -r(1 - \frac{N}{K})(1 - \frac{N}{K+1})N$$