Math 331/631 Homework Assignment #2 Due February 6

1. (a) Write a program for the secant method using the code for the bisection method (posted online) as a guide.

   (b) Use this to estimate the roots of $g(x) = x - 0.2\sin(x) + 0.5$. Include a table of errors.

2. Using your bisection program from the previous week,consider the function

$$f(x) = (x+2)(x+1)^2x(x-1)^3(x-2) \tag{1}$$

   (a) Plot the function f. Make sure to put a title, label axes, set the line width, and set the font to a readable size. What are the zeros of this function?

   (b) What zero does the bisection method converge to on the following intervals: $[-1.5, 2.5]$, $[-0.5, 2.4]$, $[-0.5, 3]$, $[-3, -0.5]$?

   (c) Explain why the bisection method converges to a particular zero when there are multiple zeros of the function on an interval.

3. Consider finding the root of $f(x) = \cos(x) - x$ in $[0, \pi/2]$. Using $p_0 = \pi/4$ and a tolerance of 0.0000001.

   (a) Find the root with Newton's method ($p_0 = 1$)

   (b) Find the root with fixed point iteration with $g(x) = \cos(x)$ ($p_0 = 1$).

   (c) Find the root with secant method starting with $[0, \pi/2]$.

   For each method, assume your last iteration is the exact solution and call it x^*. Compute the error for each iteration as $E_n = |x_n - x^*|$. Plot the points $(\log(E_{n-1}), \log(E_n))$ and estimate the slope of the line you get. It should give you the order of convergence of the method. Explain.

4. Explain what the following minimalist program does.

```
g =@(x) cos(x);      x0 = 1;
Nmax = 50;          tol = 1e-5;
x(1) = x0;
for i=2:Nmax
    x(i) = g(x(i-1));
    if abs(x(i)-x(i-1))<tol, break, end
end
format long
x = x(:)
```