Limit Sets and Applications to Foliations

Christopher Stover
Florida State University

Grad Student Seminar
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Outline

Introduction to Limit Sets

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Part I

Definitions

Definition.

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- In the case of $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, these are the linear fractional transformations
$$f(z) = \frac{az+b}{cz+d},$$
$a, b, c, d \in \mathbb{C}$ nonzero.

Notation

$$E^n \quad = \quad n\text{-dimensional Euclidean space}$$

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$$

Remark.

Clearly, there is a natural group action $M(B^n) \times B^n \to B^n$ defined by $(\varphi, x) \mapsto \varphi(x)$.

(More) Definitions

Definition.
An element $\varphi \in M(B^n)$ is:

(More) Definitions

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- **parabolic** if it fixes a unique point of S^{n-1}.

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  - $\varphi(z) = z + a$, a translation.
- **hyperbolic** if fixes two points of S^{n-1}.
  - $\varphi(z) = a^2 z$ for $a \in \mathbb{R}$, a contraction/dilation.

(Even More) Definitions

Definition.

A point $a \in S^{n-1}$ is a **limit point** of a subgroup $G \leq M(B^n)$ if there is a point $x \in B^n$ and a sequence $\{g_i\}_{i=1}^{\infty}$ of elements of G such that $g_i x \to a$ as $i \to \infty$.

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The **limit set** $L(G)$ of a subgroup $G \leq M(B^n)$ is the collection of all limit points of G.

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Definition.
The **limit set** $L(G)$ of a subgroup $G \leq M(B^n)$ is the collection of all limit points of G.

Remark.
This is a specific notion of the more general term *limit set* appearing in the study of dynamical systems and defined to be "the state of a dynamical system after an infinite amount of time."

Apollonian Gasket

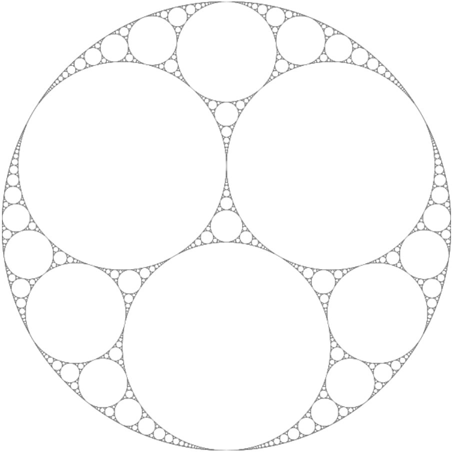

Figure 1
The Apollonian gasket is the limit of an iterated process

Part II

Dr. Curt McMullen

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Born May 21, 1958

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Education

Ph.D. from Harvard

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Current Position

Maria Moors Cabot Professor,
Harvard University

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Education
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Current Position
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Awards
Fields Medal (1998), Salem Prize (1991), etc.

The lim Program

McMullen's description in the "Read Me" file:

The LIM Program

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Limit Sets of Kleinian Groups

The program lim draws the orbits of circles under the action of a group of Möbius transformations.

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Remark.

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Remark.

- Here, we're in the case of $n = 2$.

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- $M(\mathbb{C}) \cong \mathrm{PSL}(2, \mathbb{C})$

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Limit Sets of Kleinian Groups

The program lim draws the orbits of circles under the action of a group of Möbius transformations.

Remark.

- Here, we're in the case of $n = 2$.
- $M(\mathbb{C}) \cong \mathrm{PSL}(2, \mathbb{C})$
- A **Kleinian Group** is a discrete subgroup of $\mathrm{PSL}(2, \mathbb{C})$.

How It Works—Short Version

Required Input

- Circles $c_1, \ldots, c_i$ known to be in the limit set

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Technical Input

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- Output style options

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- Circles $c_1, \ldots, c_i$ known to be in the limit set

Optional Input

- Circles $r_1, \ldots, r_k$ in which to define reflections for c_α

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- Circles $r_1, \ldots, r_k$ in which to define reflections for c_α
- Matrices $m_1, m_2, \ldots, m_j$, $t_1, \ldots, t_\ell \in \mathrm{PSL}(2, \mathbb{C})$ to be applied to the c_α and to the coordinate system, respectively

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- Circles $u_1, \ldots, u_n$ in which to define reflections of the coordinate system

How It Works—Short Version

Behind the Scenes

- LIM applies the group
  $G = \langle m_1, \ldots, m_j, r_1, \ldots, r_k \rangle$
  to the collection $C = \{c_\alpha\}$.

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- Iterates of these group actions are stored in stacks, parsed, sorted, and finalized.

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- The output is a collection of data in .ps format.

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Output

- The output is a collection of data in .ps format.
- This can be converted to visual representations in .pdf format.

Example 1

hex.run

```
./lim -d 8 -s -h 3 <<eof > hex.ps
c 0.86602540378443 8 0.0 -0.5
c 0.25 0.433012701892219 -0.166666666666
c -0.25 0.433012701892219 -0.833333333333
r 0.866025403784438 0.0 -0.5
r 0.25 0.433012701892219 -0.166666666666
r -0.25 0.433012701892219 -0.833333333333
eof
```

Graph on sphere; omit to graph in plane
Output file name
Two different threshold variables

Example 1

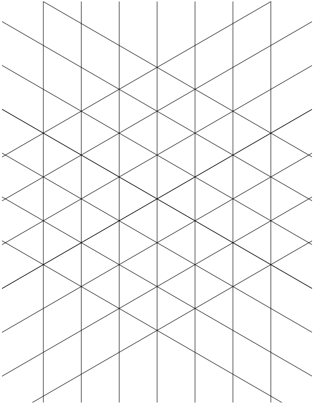

Figure 2
hex.ps without -s

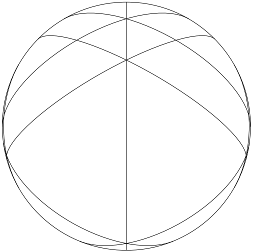

Figure 3
hex.ps with -s

Example 2

Example.run

```
./lim -s -d 60 -e 0.0001 <<eof > Example.ps
c 0.  0.  1
m 1 1 0 1 0 -1 1 -1
m 1 -1 0 -1 0 1 1 1
m 0.955 -0.025 0.045 0.025 -1.955 0.025 0.955 -0.025
m 0.955 -0.025 -0.045 -0.025 1.955 -0.025 0.955 -0.025
eof
```

A different threshold variable

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m 0.955 -0.025 -0.045 -0.025 1.955 -0.025 0.955 -0.025
eof
```

Remark.

A different threshold variable

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m 0.955 -0.025 -0.045 -0.025 1.955 -0.025 0.955 -0.025
eof
```

Remark.

According to McMullen: "This is a picture of the limit set of a Kleinian group on the boundary of Maskit's embedding of the Teichmuller space of a once-punctured torus."

A different threshold variable

Example 2

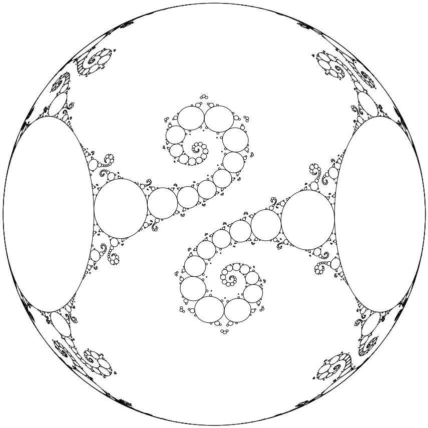

Figure 4
Example.ps

Example 3

Schottky.run

```
./lim -d 10 -e .001 <<eof > schottky2.ps
r 0 1 .7
r 0.866025 -.5 .8
r -0.866025 -.5 .8
c 0 1 .7
c 0.866025 -.5 .8
c -0.866025 -.5 .8
eof
```

Example 3

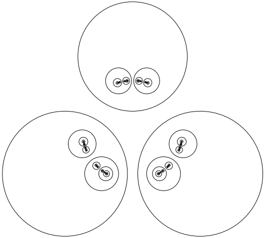

Figure 5
In the plane

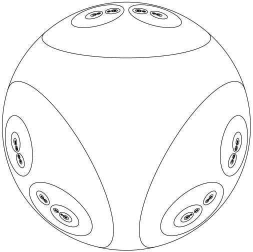

Figure 6
On the sphere

Example 4

ngon4.run

```
./lim -a 1000 -b -d 100 -e 0.001
-c 0 0 1 -w -1.1 -1.1 1.1 1.1
<<eof > ngon4.ps
r 1.553773974030037 0 1.189207115002721
c 1.553773974030037 0 1.189207115002721
r 0 1.553773974030037 1.189207115002721
c 0 1.553773974030037 1.189207115002721
r -1.553773974030037 0 1.189207115002721
c -1.553773974030037 0 1.189207115002721
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eof
```

Optional style parameter
A *different* threshold variable

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Remark.

Optional style parameter
A *different* threshold variable

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r 0 -1.553773974030037 1.189207115002721
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eof
```

Remark.
According to McMullen: "Tiling of H for torus with orbifold point of order 2."

Optional style parameter
A *different* threshold variable

Example 4

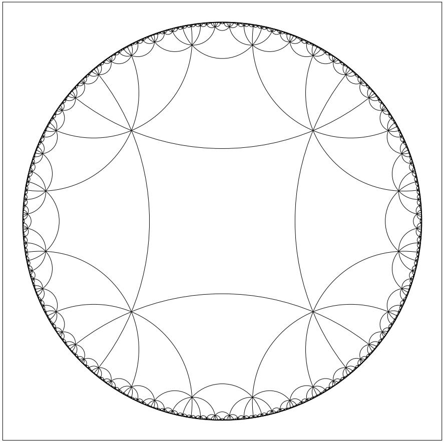

Figure 7

`ngon4.ps` + a box because of -b

Example 5

lattice.run

```
./lim -s -d 10 <<eof > lattice.ps
c 0 0 0.0
c 0 0 -.5
m 1 0 1 0 0 0 1 0
m 1 0 0 1 0 0 1 0
m 1 1 0 0 0 0 1 0
u .3 .4 2
eof
```

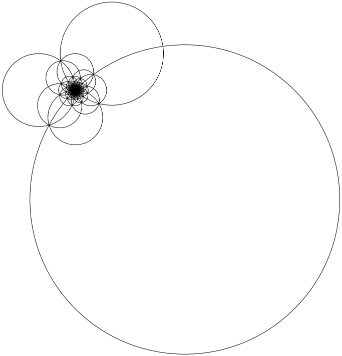

Figure 8
In the plane

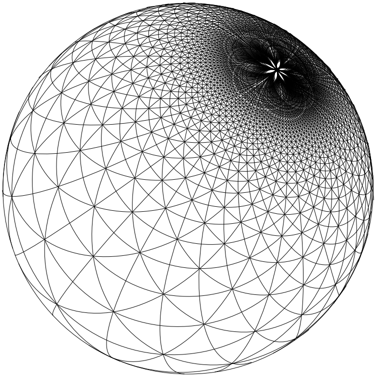

Figure 9
On the sphere

Part III

Introduction

Definition (Foliations, Loosely).

A *foliation* of a differentiable manifold M^n is a decomposition of M into connected submanifolds of dimension k which stack up locally like subsets of $\mathbb{R}^n = \mathbb{R}^k \times \mathbb{R}^{n-k}$.

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- The k-submanifolds in the decomposition are called *leaves*.

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- The k-submanifolds in the decomposition are called *leaves*.
- k is known as the dimension of the foliation.
- $n - k$ is its codimension.
- There are some "compatibility conditions."

Examples

Ex 1. $\mathbb{R}^n$ foliated by k-planes, i.e. $\mathbb{R}^n = \mathbb{R}^k \times \mathbb{R}^{n-k}$.

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Ex 2. $\mathbb{R}^2$ foliated by contours of $f(x, y) = (x^2 - 1)e^y$.

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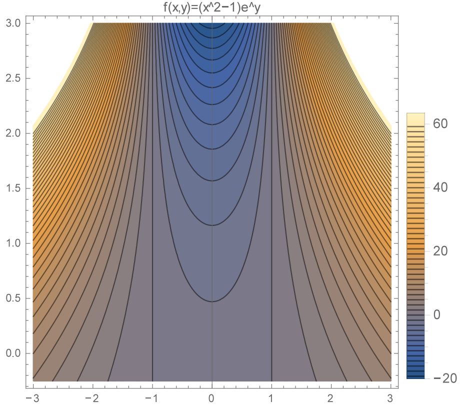
Ex 2. $\mathbb{R}^2$ foliated by contours of $f(x, y) = (x^2 - 1)e^y$.



Examples

Ex 3. $\mathbb{R}^3$ foliated by surfaces of revolution.

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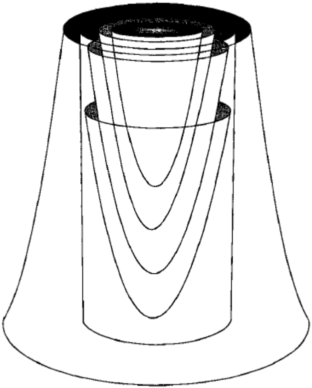

Examples

Ex 4. Reeb foliation of $D^2 \times S^1$.

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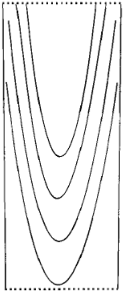

Examples

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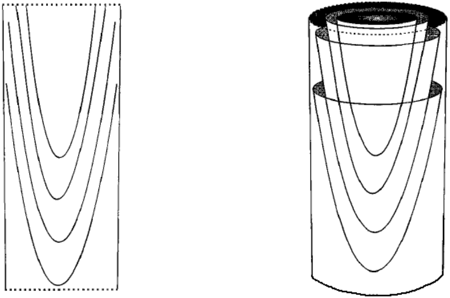

Examples

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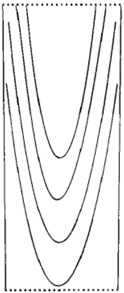
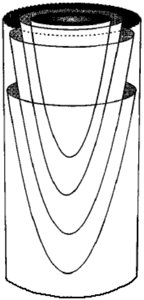
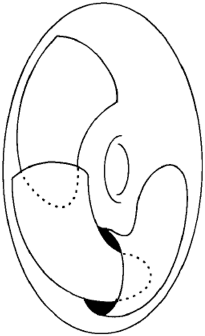
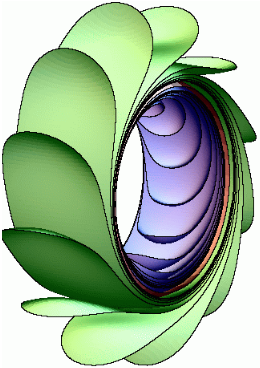
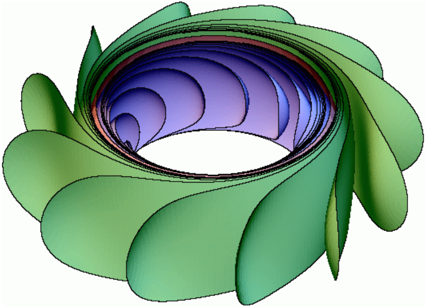






Reeb Foliation (Cont'd)





Facts About Foliated 3-Manifolds

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- Both of these ideas may be relevant when talking about the limit sets of foliations.

Big Idea

Is it possible to modify the above results in order to get decent pictures of the limit sets of the lifts of so-called “finite-depth foliations” to the universal cover of hyperbolic 3-manifolds?

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- Win a Fields medal. :)

Thank you!