MATH FUN DAY

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Generating Very Large Numbers

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POWERS OF 10

$$10^N = \underbrace{10\cdots\cdots0}_{N}$$ 1 followed by N zeros

1

10

$10^2 = 100$

$10^3 = 1{,}000$

$10^4 = 10{,}000$

$10^5 = 100{,}000$

10^6 - 1 million

10^9 - 1 billion

10^{12} - 1 trillion

10^{15} - 1 quadrillion

...

$10^{100} = 1$ GOOGOL

$10^{303} = 1$ CENTILLION

$10^{\text{GOOGOL}} = 10^{10^{100}} = 1$ GOOGOLPLEX

BIG NUMBERS

GAZILLIONS AROUND US

13.79 BILLION YRS. = 13.79×10^9 YRS. Age of the universe

93×10^9 LIGHT YRS Diameter of the Universe

10^{80} Estimated number of particles

More than 10^{14} Cells in the human body

1.6×10^{18} bits Total amount of printed material

Guess What Comes Next...

Sequences

- 1, 1, 2, 3, 5, 8, 13, 21, ...

- 1, 1, 2, 6, 24, 120, 720, ...

- 1, 2, 720!, ...

- 1, 1, 2, 3, 5, 8, 13, 21, 34
- 1, 1, 2, 6, 24, 120, 720, ...
- 1, 2, 720!, ...

▷ 1, 1, 2, 3, 5, 8, 13, 21, 34

$3! = 3\cdot 2$ $\quad 4! = 4\cdot 6$ $\quad 5! = 5\cdot 24$ $\quad 6! = 6\cdot 120$ $\quad 7! = 7\cdot 720$

▷ 1, 1, 2, 6, 24, 120, 720, 5040

$2\cdot 1 = 2!$

▷ 1, 2, 720!, ...

▷ $1, 1, 2, 3, 5, 8, 13, 21, 34$

$3! = 3\cdot 2$, $4! = 4\cdot 6$, $5! = 5\cdot 24$, $6! = 6\cdot 120$, $7! = 7\cdot 720 = 5040$

▷ $1, 1, 2, 6, 24, 120, 720, 5040$

$2\cdot 1 = 2!$

▷ $1, 2, 720!, 4!!!!$!

$1 = 1!$, $2 = 2! = 2!!$, $720! = 6!! = 3!!!$

GUESS WHAT

COMES NEXT...

OPERATIONS

BEYOND POWERS

$$a^b = \underbrace{a \times a \times \cdots \times a}_{b} = a \uparrow b$$

$3^4 = 3 \times 3 \times 3 \times 3$, $2^5 = 2 \times 2 \times 2 \times 2 \times 2$, ...

Powers of 10, ...

$$a \uparrow\uparrow b = a^{a^{a^{\cdot^{\cdot^{\cdot}}}}}$$ There are b a's. $$= a \uparrow a \uparrow \cdots \uparrow a$$

$3 \uparrow\uparrow 3 = 3^{3^3} = 7625597484987$, $2 \uparrow\uparrow 4 =$?, $2 \uparrow\uparrow 5 =$?

There are b a's.

$$a \uparrow\uparrow\uparrow b = a \uparrow\uparrow a \uparrow\uparrow \cdots \uparrow\uparrow a$$

$$2 \uparrow\uparrow\uparrow 4 = 2 \uparrow\uparrow 2 \uparrow\uparrow 2 \uparrow\uparrow 2 = 2^{2^{2^{\cdot^{\cdot^{\cdot}}}}}$$ How many?

BEYOND POWERS

$$a^b = \underbrace{a \times a \times \cdots \times a}_{b} = a \uparrow b$$

$3^4 = 3 \times 3 \times 3 \times 3$, $2^5 = 2 \times 2 \times 2 \times 2 \times 2$, ...

Powers of 10, ...

$$a \uparrow\uparrow b = a^{a^{a^{\cdot^{\cdot^{\cdot}}}}}$$

There are b a's.

$$= a \uparrow a \uparrow \cdots \uparrow a$$

$3 \uparrow\uparrow 3 = 3^{3^3} = 7625597484987$, $2 \uparrow\uparrow 4 =$?, $2 \uparrow\uparrow 5 =$?

There are b a's.

$$a \uparrow\uparrow\uparrow b = a \uparrow\uparrow a \uparrow\uparrow \cdots \uparrow\uparrow a$$

$$2 \uparrow\uparrow\uparrow 4 = 2 \uparrow\uparrow 2 \uparrow\uparrow 2 \uparrow\uparrow 2 = 2^{2^{2^{\cdot^{\cdot^{\cdot}}}}}$$

How many?

65536 !

THE LAST (?) SEQUENCE

$n = 1$ $\qquad$ $n = 2$ $\qquad$ $n = 3$ $\qquad$ $n = 4$

$$1 \uparrow 1 \qquad 2 \uparrow\uparrow 2 \qquad 3 \uparrow\uparrow\uparrow 3 \qquad 4 \uparrow\uparrow\uparrow\uparrow 4$$

$$1 \uparrow 1 = 1$$

$$2 \uparrow\uparrow 2 = 4$$

$$3 \uparrow\uparrow\uparrow 3 = 7625597484987$$

CAN'T COMPUTE !

Now we know of much bigger numbers .....

SO THE STORY DOES NOT END ...