

MATH
FUN DAY



Florida State University Department of Mathematics

GENERATING
VERY LARGE
NUMBERS

OCTOBER 11, 2014

ETTORE ALDROVANDI (MATHEMATICS)

POWERS OF 10

$$10^N = \underbrace{10 \dots\dots 0}_N \quad 1 \text{ followed by } N \text{ zeros}$$

1

10

$$10^2 = 100$$

$$10^3 = 1,000$$

$$10^4 = 10,000$$

$$10^5 = 100,000$$

$$10^6 - 1 \text{ million}$$

$$10^9 - 1 \text{ billion}$$

$$10^{12} - 1 \text{ trillion}$$

$$10^{15} - 1 \text{ quadrillion}$$

...

$$10^{100} = 1 \text{ GOOGOL}$$

$$10^{303} = 1 \text{ CENTILLION}$$

$$10^{\text{GOOGOL}} = 10^{10^{100}} = 1 \text{ GOOGOLPLEX}$$

BIG NUMBERS

GAZILLIONS AROUND US

13.79 BILLION YRS. = 13.79×10^9 YRS. Age of the universe

93×10^9 LIGHT YRS Diameter of the Universe

10^{80} Estimated number of particles

More than 10^{14} Cells in the human body

1.6×10^{18} bits Total amount of printed material

GUESS WHAT

COMES NEXT...

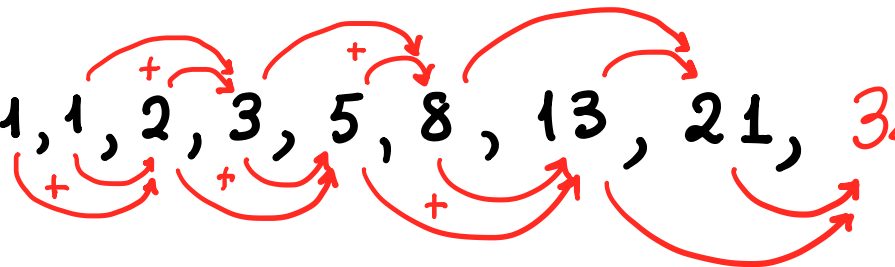
SEQUENCES

▷ 1, 1, 2, 3, 5, 8, 13, 21, ...

▷ 1, 1, 2, 6, 24, 120, 720, ...

▷ 1, 2, 720!, ...

▷ 1, 1, 2, 3, 5, 8, 13, 21, 34



The diagram illustrates the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34. Red curved arrows with a '+' sign connect the two preceding terms to the next term. For example, an arrow from the first '1' to the second '1' with a '+' sign points to the '2'. This pattern continues for all subsequent terms, showing that each term is the sum of the two terms immediately before it.

▷ 1, 1, 2, 6, 24, 120, 720, ...

▷ 1, 2, 720!, ...

▷ 1, 1, 2, 3, 5, 8, 13, 21, 34

The diagram illustrates the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34. Red curved arrows with '+' signs connect pairs of consecutive terms to their sum: (1, 1) → 2, (1, 2) → 3, (2, 3) → 5, (3, 5) → 8, (5, 8) → 13, (8, 13) → 21, and (13, 21) → 34.

▷ 1, 1, 2, 6, 24, 120, 720, 5040

The diagram illustrates the factorial sequence: 1, 1, 2, 6, 24, 120, 720, 5040. Red curved arrows show the multiplication of previous terms to get the next term: (1, 1) → 2, (2, 2) → 6, (6, 3) → 24, (24, 5) → 120, (120, 6) → 720, and (720, 7) → 5040. Above each term from 6 onwards, its factorial representation is shown: 3! above 6, 4! above 24, 5! above 120, 6! above 720, and 7! above 5040. Double lines connect the factorial term to the product of the previous term and the index: 3! = 3 · 2, 4! = 4 · 6, 5! = 5 · 24, 6! = 6 · 120, and 7! = 7 · 720. Below the first arrow, 2! is shown as 2 · 1.

▷ 1, 2, 720!, ...

▷ 1, 1, 2, 3, 5, 8, 13, 21, 34

The diagram shows the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34. Red curved arrows with a '+' sign connect the two terms before each term to show the addition process: 1+1=2, 1+2=3, 2+3=5, 3+5=8, 5+8=13, 8+13=21, and 13+21=34.

▷ 1, 1, 2, 6, 24, 120, 720, 5040

The diagram shows the factorial sequence: 1, 1, 2, 6, 24, 120, 720, 5040. Red curved arrows show the multiplication process: 1 → 1 (labeled 1!), 1 → 2 (labeled 2!), 2 → 6 (labeled 3! = 3 · 2), 6 → 24 (labeled 4! = 4 · 6), 24 → 120 (labeled 5! = 5 · 24), 120 → 720 (labeled 6! = 6 · 120), and 720 → 5040 (labeled 7! = 7 · 720). The factorial notation is shown above each term, and the multiplication step is shown below the arrow.

▷ 1, 2, 720!, 4!!!!

The diagram shows the hyperfactorial sequence: 1, 2, 720!, 4!!!!. Red curved arrows show the multiplication process: 1 → 2 (labeled 1! and 2!), 2 → 720! (labeled 2!! and 3!!!), and 720! → 4!!!! (labeled 6!! and 3!!!!). The factorial notation is shown below each term, and the multiplication step is shown above the arrow.

GUESS WHAT

COMES NEXT...

OPERATIONS

BEYOND POWERS

$$a^b = \underbrace{a \times a \times \dots \times a}_b = a \uparrow b$$

$$3^4 = 3 \times 3 \times 3 \times 3, 2^5 = 2 \times 2 \times 2 \times 2 \times 2, \dots$$

Powers of 10, ...

$$a \uparrow \uparrow b = a^{a^{a^{\dots}}} \quad \text{There are } b \text{ } a\text{'s.} \quad = a \uparrow a \uparrow \dots \uparrow a$$

$$3 \uparrow \uparrow 3 = 3^{3^3} = 7625597484987, \quad 2 \uparrow \uparrow 4 = ?, \quad 2 \uparrow \uparrow 5 = ?$$

$$a \uparrow \uparrow \uparrow b = a \uparrow \uparrow a \uparrow \uparrow \dots \uparrow \uparrow a \quad \text{There are } b \text{ } a\text{'s.}$$

$$2 \uparrow \uparrow \uparrow 4 = 2 \uparrow \uparrow 2 \uparrow \uparrow 2 \uparrow \uparrow 2 = 2^{2^{2^2}} \quad \text{How many?}$$

BEYOND POWERS

$$a^b = \underbrace{a \times a \times \dots \times a}_b = a \uparrow b$$

$$3^4 = 3 \times 3 \times 3 \times 3, 2^5 = 2 \times 2 \times 2 \times 2 \times 2, \dots$$

Powers of 10, ...

$$a \uparrow \uparrow b = a^{a^{a^{\dots}}} \quad \text{There are } b \text{ } a\text{'s.} \quad = a \uparrow a \uparrow \dots \uparrow a$$

$$3 \uparrow \uparrow 3 = 3^{3^3} = 7625597484987, \quad 2 \uparrow \uparrow 4 = ?, \quad 2 \uparrow \uparrow 5 = ?$$

$$a \uparrow \uparrow \uparrow b = a \uparrow \uparrow a \uparrow \uparrow \dots \uparrow \uparrow a \quad \text{There are } b \text{ } a\text{'s.}$$

$$2 \uparrow \uparrow \uparrow 4 = 2 \uparrow \uparrow 2 \uparrow \uparrow 2 \uparrow \uparrow 2 = 2^{2^{2^{2^{\dots}}}} \quad \text{How many?}$$

65536 !

THE LAST(?) SEQUENCE

$n=1$

1 ↑ 1

"
1

$n=2$

2 ↑↑ 2

"
4

$n=3$

3 ↑↑↑ 3

"
7625597484987

$n=4$

4 ↑↑↑↑ 4

CAN'T COMPUTE!

Now we know of much bigger numbers

SO THE STORY DOES NOT END ...