2.7 Preparation

Before we cover 2.7 in class read this section, 2.7 Preparation, of the course notes and complete *2.7 Preparation* on WebAssign following the instructions in the order in which they are given are given below.

1. 2.7 Recall how to find slopes of lines and average velocity

A line secant to a curve is simply a line through two distinct points on the curve. The word secant here has nothing to do with the word secant in trigonometry. Fill in the following formulas.

(1) The slope of the secant line of $y = f(x)$ through $(a, f(a))$ and $(b, f(b))$ is

$m_{sec} =$

(2) The average velocity of a particle with position at time t given by $s(t)$ over the time interval $[a, b]$ is

$v_{ave} =$

Compare the formula for the average rate of change below to the formulas above. Note that the slope of the secant and the average velocity are examples of average rates of change.

Definition 1.1. *The* **Average Rate of Change** *of* $y = f(x)$ *from the point* $(a, f(a))$ *to* $(b, f(b))$ *(or over the interval* $[a, b]$*) is* $\dfrac{f(b) - f(a)}{b - a}$.

Compare the formula for the average rate of change to the formula for the derivative given below.

Definition 1.2. *The* **Instantaneous Rate of Change** *or the* **Derivative** *of* $y = f(x)$ *at* $(a, f(a))$ *is* $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$.

Continued on the next page.

2.7 Preparation Continued

Now go to *2.7 Preparation* on WebAssign.

(1pt) WebAssign *2.7 Preparation* #1(2.7.EI.002). Use the "Explore and Test" to explore how the slope changes as you move the points. In particular, as the two points get closer together notice that the secant line approaches a tangent line. Then answer the questions. The video will provide further insight into derivatives vs average rate of change.

(1pt) WebAssign *2.7 Preparation* #2(2.7.018). Use definition 1.1.

(1pt) WebAssign *2.7 Preparation* #2(2.7.047). For part (b) just use the average rate of change in part (a) that should be closest to make the estimate.

2.7 Preparation End

2. Definition of a Derivative

Definition 2.1. *For the following we assume $f(x) = y$ is a function.*

The **derivative of f with respect to x** *at $x = a$ is*

$$f'(a) = y'(a) = \frac{d}{dx}f(x)\Big|_{x=a} = \frac{dy}{dx}\Big|_{x=a} = Df(a) = D_x f(a) =$$

Example 2.1. *Find the derivative of $f(x) = \dfrac{4}{2-x}$ at $x = 0$.*

Example 2.2. *Find* $f'(a)$.

$$f(x) = \frac{4}{2-x}$$

Example 2.3. *(2.7 WebAssign Homework #7(2.7.035)) Find* $f'(a)$ *if* $f(x) = \sqrt{1-4x}$.

Remark 2.1. *All of the following concepts are found using the derivative:*

(1) the slope of a tangent line,
(2) velocity of a particle using the position,
(3) the acceleration of a particle using velocity,
(4) instantaneous rate of change of a quantity
(5) marginal cost using a cost function
(6) marginal revenue using a revenue function

3. Examples

Example 3.1. *(2.7 WebAssign Homework #5(2.7.029))*

(1) If $F(x) = \frac{5x}{1+x^2}$ *find* $F'(2)$ *and use it to find an equation of the tangent line to the curve* $y = \frac{5x}{1+x^2}$ *at the point* $(2, 2)$.
(2) Select the graphs of F *(on the WebAssign problem) with the tangent at* $(2, 2)$.

Example 3.2. *(2.7 WebAssign Homework #2(2.7.013.MI))*

If a ball is thrown into the air with a velocity of 33 ft/s, its height (in feet) after t seconds is given by $y = 33t - 16t^2$. Find the velocity when $t = 1$.

Example 3.3. *State the function and value for which the following is its derivative at a.*

$$\lim_{x \to \pi/4} \frac{\tan x - 1}{x - \pi/4}$$

Example 3.4. *State the function and value for which the following is its derivative at a.*

$$\lim_{h \to 0} \frac{\sqrt[3]{27 + h} - 3}{h}$$

2.7 Homework

Deadline for WebAssign Homework assignments are 8am on the due date given on Canvas. **You are expected to bring an internet capable device and while participating in lecture you will submit some of the homework in class. The rest of the problems should be worked on after the section is covered in lecture and before the next class.**

Deadline for Written Exercises (if any) are the beginning of the period in recitation.

"Drill" problems are the type that you need to become almost automatic. "Putting it together" problems are problems use what we have learned from this section and prior sections and/or require more in-depth thought about how to apply the concepts. Not every section will have both types of exercises. Work problem in the following order for best comprehension and mastery.

Drill Exercises:
(0pt) 2.7 WebAssign Homework # 1(2.7.003),
(1pt) 2.7 WebAssign Homework #2(2.7.013.MI)
(0pt) 2.7 WebAssign Homework # 3(2.7.015)
Putting it together:
(1pt) 2.7 WebAssign Homework # 4(2.7.014)
Drill Exercises:
(3pt) 2.7 WebAssign Homework #5(2.7.029), 6(2.7.033), 7(2.7.035)
(0pt) 2.7 WebAssign Homework # 8(2.7.041)
(1pt) 2.7 WebAssign Homework # 9(2.7.044)
Putting it together:
(2pt) 2.7 WebAssign Homework # 10(2.7.020), 11(2.7.057)
(1pt) 2.7 WebAssign Homework # 11: Written Exercise from text # 24. Write the solution to this exercise on a piece of paper to be turned in at the beginning of recitation. Your name and section number should be clearly written at the top right of the page.