4.3 Preparation

Recall from section 2.8 our discussion of how the graph of $y = f(x)$ and the graph of $y = f'(x)$ were related. In this section we go further into the relationships in the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$.

(0pt) WebAssign *4.3 Preparation* #1(1.1Inc/DecDefs) The first step is to just review what we mean by increasing, decreasing, concave up, and concave down on a graph. Review the definitions below. If necessary, review the definitions in Section 1.1 of the text.

1. 4.3 Increasing and Decreasing

Definitions 1.1. *Let f be a continuous function on the interval (a, b).*

(1) We say f is **increasing** *on (a, b) if for any two values, x_1 and x_2 in the interval (a, b) with $x_1 < x_2$ we have $f(x_1) < f(x_2)$.*
(2) We say f is **decreasing** *on (a, b) if for any two values, x_1 and x_2 in the interval (a, b) with $x_1 < x_2$ we have $f(x_1) > f(x_2)$.*

(1pt) Solve parts (a) and (b) of WebAssign *4.3 Preparation* #2(SCalcET8 4.3.001). The remaining parts should be solved after going over the definitions of concavity.

Remark 1.1. *Sometimes* **increasing** *and* **strictly increasing** *are defined slightly differently and in such a way that a constant function would be considered both increasing and decreasing, but not strictly increasing/decreasing. However, in our class, we will mean the same for increasing and strictly increasing and will consider a* **constant** *function to be neither increasing nor decreasing.*

Continue on the next page.

2. 4.3 Concavity

Definitions 2.1. *Let f be a continuous function on the interval (a, b) and assume f is twice differentiable everywhere in the interval (a, b).*

(1) f is **concave up** *on the interval (a, b) if the graph of $y = f(x)$ lies above the line tangent to the graph at any point in the interval. Visually, a concave up shape is like the shape of a bowl that holds water.*
(2) f is **concave down** *on the interval (a, b) if the graph of $y = f(x)$ lies below the line tangent to the graph at any point in the interval. Visually, a concave down shape is like the shape of a bowl that is upside down.*
(3) If f changes concavity at $x = c$ then we say $(c, f(c))$ is a **inflection point***.*

(0pt) Use Animation: Figure 4.3.006a and Animation: Figure 4.3.006b in the e-text to understand how to visualize what the above definitions are saying.
(1pt cont.) Now finish *4.3 Preparation* #2(SCalcET8 4.3.001) #1(SCalcET8 4.3.001).

3. The shape of f v.s. f'

(1pt) WebAssign *4.3 Preparation* #3(4.3IncDecThm) Fill in the blanks below.

Now we move on to how the shape of the graph of $y = f(x)$ can be determined by f' and f''. Fill in the blanks in the following Theorem by referring to the theorems near the end of the section 4.2 notes.

Theorem 3.1. *Suppose f is a function that is differentiable on the interval (a, b).*

1. $f'(x) = 0$ for all x in the interval (a, b) if and only if f is a ______________________ function on (a, b).
2. $f'(x) > 0$ for all x in the interval (a, b), except possibly a finite number of points, if and only if f is a ______________________ function on (a, b).
3. $f'(x) < 0$ for all x in the interval (a, b), except possibly a finite number of points, if and only if f is a ______________________ function on (a, b).

(1pt) WebAssign *4.3 Preparation* #4(SCalcET8 4.3.009). Work part (a) only. Use the above theorem and the video if necessary. The topics in the other parts will be discussed in class.

4.3 Preparation End

Example 3.1. *4.3 WebAssign Homework (SCalcET4.3.015)* $f(x) = e^{5x} + e^{-x}$

(1) Find all intervals where the function is increasing and decreasing.

Remark 3.1. *In our class we will not check whether or not to include the end points and will always use open intervals for our solutions for the intervals on which a function is increasing and/or decreasing, since this is how our text book handles intervals. However, in other texts the endpoints may be considered and in example In example 8.1, the function is increasing on the closed intervals.*

4. First Derivative Test for Local Extrema

First Derivative Test:

Find all critical numbers of f. Keep in mind that all critical numbers must be in the domain of f.

1. If f' is positive to the left of c and negative to the right of c, then f has a...

2. If f' is negative to the left of c and positive to the right of c, then f has a...

3. If f' does not change signs at c, then f...

5. Finding **LOCAL** extrema using the First Derivative Test

1. Find the domain of f.
2. Find all critical numbers of f. These will be your potential extrema.
3. Place all critical numbers AND values where f is undefined on a number line. These numbers will separate the number line into intervals.
4. Determine the sign of f' on each interval on the number line.
5. Use the information in 4 to determine intervals where f is increasing, decreasing, and where local extremes occur.

Example 3.1 continued (4.3 WebAssign Homework (SCalcET4.3.015)) $f(x) = e^{5x} + e^{-x}$

(2) Find the local extrema of f.

Example 5.1. *Sketch a graph of the function given the following.*

f is continuous on the domain $(-\infty, 1) \cup (1, \infty)$, $\lim_{x \to \pm\infty} f(x) = 3$, $\lim_{x \to 1} |f(x)| = \infty$, $f(-2) = -2$, $f(4) = 6$, $f'(-2) = 0$, $f'(x) > 0$ *on* $(-2, 1)$, *and* $f'(x) < 0$ *on* $(-\infty, -2) \cup (1, \infty)$.

6. The shape of f v.s. f''

Theorem 6.1. *Suppose f is a function that is twice differentiable on the interval (a, b).*

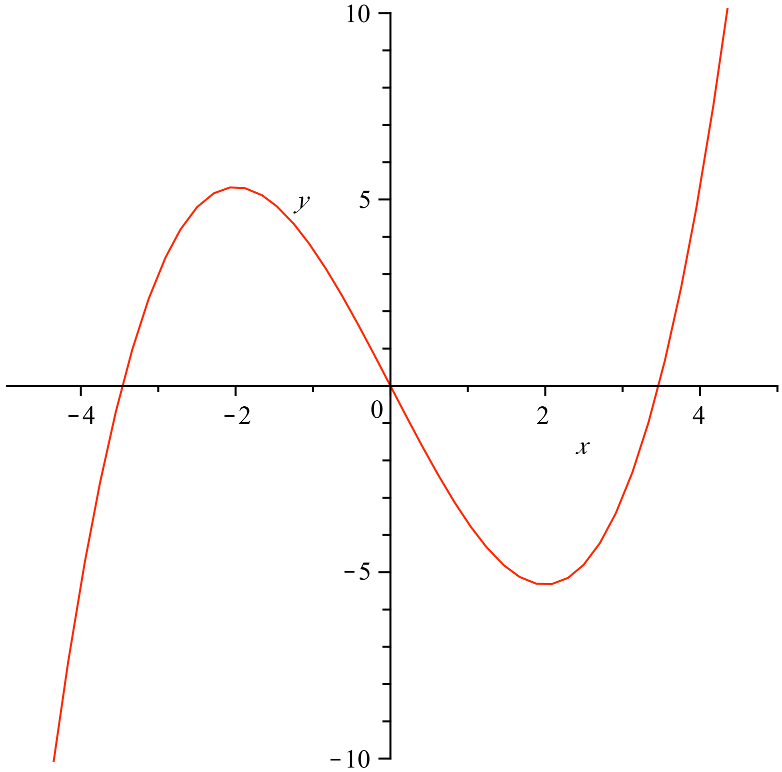
1. *$f''(x) = 0$ for all x in the interval (a, b) if and only if f is a linear function on (a, b).*
2. *$f''(x) > 0$ for all x in the interval (a, b), except possibly a finite number of points, if and only if f is concave up on (a, b).*
3. *$f''(x) < 0$ for all x in the interval (a, b), except possibly a finite number of points, if and only if f is concave down on (a, b).*

Example 3.1 continued (4.3 WebAssign Homework (SCalcET4.3.015)) $f(x) = e^{5x} + e^{-x}$

(3) Find all intervals where the function is concave up and concave down. Find all inflection point(s).

Remark 6.1. *The prior theorems tell us that we can use the sign of the first derivative determines where a function is increasing and decreasing, while the sign of f'' determines the concavity of f.*

Example 6.1. *Suppose the graph below is the graph of $y = f'(x)$ given that $f(0) = 0$. Sketch a possible graph for $y = f(x)$. Make local extrema, increasing, decreasing, and concavity clear.*



7. The Second Derivative Test for Local Extrema

Theorem 7.1 (Second Derivative Test)**.** *Suppose $y = f(x)$ is such that $f'(c) = 0$ (and f is twice differentiable around c).*

(1) If $f''(c) > 0$ then ____

(2) If $f''(c) < 0$ then ____

(3) If $f''(c) = 0$ then ____

Example 7.1. *Can the second derivative test be used to find all the local extrema of the following function? Find any local extrema the second derivative test can be used for and find the rest using the first derivative test.*

$h(x) = 4x^5 - 5x^4 - 40x^3$

8. Graphing

We can put together all the information we know so far to create graphs that show the right shape in addition to the information we could find before (like domain and asymptotes).

Example 8.1. *4.3 WebAssign Homework (SCalcET4.3.056) Consider the function* $f(x) = e^{\arctan(5x)}$.

(1) Find the asymptote(s).
(2) Find the interval where the function is increasing or decreasing.
(3) Find the local extreme values.
(4) Find the interval where the function is concave up or down and inflection points.
(5) Sketch the graph.

4.3 Homework

Putting it together:

(0pt) WebAssign Homework # 1(4.3.005), 2(Text 4.3 # 7), 3(4.3.009)
(2pt) WebAssign Homework # 4(4.3.014), 5(4.3.015)
(0pt) WebAssign Homework # 6(4.3.016), 7(Text 4.3 # 21)
(1pt) WebAssign Homework # 8(4.3.022)
(0pt) WebAssign Homework # 9(4.3.034)
(2pt) WebAssign Homework # 10(4.3.036), 11(4.3.045)
(0pt) WebAssign Homework # 12(4.3.047)
(2pt) WebAssign Homework # 13(4.3.506.XP), 14(4.3.053)
(2pt) WebAssign Homework # 15(4.3.056)
(1pt) WebAssign Homework # 16: Written Exercise from text 4.3 # 28