6.3 Preparation

1. 6.3 CYLINDRICAL SHELLS

In this section we will learn another method for finding the volume of a solid of revolution. We begin with the a region that will be revolved about an axis and that will create the solid that we want the volume for.

Look at the figure below on the left. The region we wish to rotate is under the red curve, $y = f(x)$, above the $x$-axis, and in the interval $0 \leq x \leq 1$. The figure on the left shows a diagram of a typical "approximating shell" that we will be using in this new method. Think of a shell as a thickened cylinder obtained by rotating the approximating rectangle that is shaded in blue about the $y$-axis. We can estimate the area of the shell by "unrolling" the shell and using that the shape is then close to a rectangular solid, pictured on the right. We can then find the volume of this rectangular solid using our favorite volume formula: $\text{length} \times \text{width} \times \text{height}$. Note that the length of the rectangular solid is the circumference of the shell.

(1pt) Solve WebAssign 6.3 Homework Before Class #1(6.3SapproxShell1). (The function whose graph is given above is not the same as the function in your WebAssign problem.)

6.3 Preparation End
2. Finding volume using cylindrical shells

If the approximating rectangle for the region is vertical (so width is $\Delta x$), then

$$V = \int_a^b \text{(circumference)}(\text{height}) \, dx,$$

where circumference and height are in terms of $x$ and $a \leq x \leq b$.

Example 2.1. WebAssign 6.3 WebAssign Homework (6.3.002ExactVol) Let $S$ be the solid obtained by rotating the region below the curve $y = 6 \sin(3x^2)$, over the interval $[1, \sqrt{\pi/3}]$, about the y-axis. Use shells to find the volume $V$ of $S$. 
If the approximating rectangle for the region is horizontal (so width is $\Delta y$), then

$$V = \int_c^d (\text{circumference})(\text{height}) \, dy,$$

where circumference and height are in terms of $y$ and $c \leq y \leq d$.

**Example 2.2.** WebAssign 6.3 WebAssign Homework (SCalcET8 6.3.009) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the $x$-axis.

$$xy = 7, \; x = 0, \; y = 7, \; \text{and} \; y = 9.$$ 

**3. Lines other than the $x$ and $y$-axis for the Axis of Rotation**

**Example 3.1.** WebAssign 6.3 WebAssign Homework (SCalcET8 6.3.019.) Use the method of cylindrical shells to find the volume $V$ generated by rotating the region bounded by the given curves about the specified axis.

$$x = 8y^2, \; y \geq 0, \; x = 8; \; \text{about} \; y = 2.$$
4. Surface of Revolution - Disc, Washer, and Shell Method Summary

Let $R$ be a region that lies entirely on one side of a line, $L$. We may revolve the region $R$ about the line $L$ to obtain a solid of revolution. Each point of $R$ is revolved about $L$ so that the point always stays the same distance from $L$, creating a circle with center on $L$ and radius the distance from $L$ to that point being revolved.

<table>
<thead>
<tr>
<th>$L$ is horizontal</th>
<th>$y = f(x)$</th>
<th>$x = f(y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc or Washer method</td>
<td>$\int_{x=-\infty}^{x=\infty} \pi (radius)^2 , dx$</td>
<td>$\int_{y=-\infty}^{y=\infty} 2\pi (rad)(ht) , dy$</td>
</tr>
<tr>
<td>$\int_{x=-\infty}^{x=\infty} \pi (outter rad)^2 , dx$</td>
<td>$\int_{y=-\infty}^{y=\infty} -\pi (inner rad)^2 , dx$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L$ is vertical</th>
<th>Shell Method</th>
<th>Disc or Washer method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\int_{x=-\infty}^{x=\infty} 2\pi (rad)(ht) , dx$</td>
<td>$\int_{y=-\infty}^{y=\infty} \pi (radius)^2 , dy$</td>
<td></td>
</tr>
<tr>
<td>$\int_{y=-\infty}^{y=\infty} \pi (outter rad)^2 , dy$</td>
<td>$\int_{y=-\infty}^{y=\infty} -\pi (inner rad)^2 , dy$</td>
<td></td>
</tr>
</tbody>
</table>

**Example 4.1.** Set up an integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate. $y = \ln x$, $y = 0$, $x = e^2$ about $x = 1$
Example 4.2. Set up another integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate. \( y = \ln x, y = 0, x = e^2 \) about \( x = 1 \)

Example 4.3. Set up an integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate. \( y = \ln x, y = 0, x = e^2 \) about \( y = 2 \)
6.3 Volumes by Cylindrical Shells

6.3 Homework

Putting it together:
(1pt) WebAssign Homework # 1(6.3.002ExactVol)
(0pt) WebAssign Homework # 2(6.3.001)
(1pt) WebAssign Homework # 3(6.3.004)
(0pt) WebAssign Homework # 4(6.3.005.MI)
(5pt) WebAssign Homework # 5(6.3.008), 6(6.3.009), 7(6.3.012), 8(6.3.015), 9(6.3.019)
(0pt) WebAssign Homework # 10(6.3.021), 11(6.3.023), 12(6.3.025)
(2pt) WebAssign Homework # 13(6.3.038), 14(6.3.042)