Homework 1 Introduction to Computational Finance Spring 2023

Solutions due Friday, 2/3/23

Answers to the homework problems should be submitted using the class canvas page.

You should submit pdf files. **Do not sent Word files or any other text processing tool's input file**.

As with all homework assignments you are allowed and encouraged to consult the relevant literature. You are also expected **to cite all literature that is used to generate your solutions and your solutions must make clear your understanding of the work cited.**

Programming Assignment

There is no programming assignment in this graded homework set.

Written Exercises

Problem 1.1

Suppose you have 4 bits to encode integers. Create a table with the unsigned binary codewords in the first column, the corresponding values represented by the codewords using 4-bit bias coding in the second column, and the corresponding values represented by the codewords using 2's complement coding in the third column.

Problem 1.2

1.2.a. How many bits does a word containing a floating point number in the system $\mathcal{F}(\beta, t, L, U)$ with $\beta = 9$, $t = 5$, $L = -7$, $U = 8$?

1.2.b. Express the following numbers as floating point numbers in the system $\mathcal{F}(\beta, t, L, U)$ with $\beta = 9$, $t = 5$, $L = -7$, $U = 8$.

- 122.9572
- 457932
- 0.0014973

1.2.c. Calculate the relative error for each number and verify it satisfies the bounds implied by the floating point system used.

1.2.d. Consider the IEEE single precision floating point encoding of a normalized floating point number:

$$\left[\ \sigma \mid \epsilon \mid \mu\ \right] = \left[\ 0 \mid 10000101 \mid 11101000000000000000000\ \right]$$

What is the number expressed as a decimal?

Problem 1.3

(1.3.a) Show by example that floating point rounded and chopped arithmetic are not associative, i.e., $a + (b + c)$ and $(a + b) + c$ are not necessarily equal if a, b, and c are floating point numbers and the computation is performed using floating point arithmetic.

(1.3.b) Is floating point arithmetic distributive?

(1.3.c) Suppose $x \in \mathbb{R}$ and $y \in \mathbb{R}$ with $x < y$. Is it always true that $fl(x) < fl(y)$ in any standard model floating point system?

Problem 1.4

Let $x \in \mathbb{R}^n$, and $y \in \mathbb{R}^n$ be two vectors with

$$x = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{pmatrix}, \quad y = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{pmatrix}.$$

$$|\xi_i| \geq 1 \qquad |\eta_i| \geq 1$$

Consider the evaluation of the two inner products

$$\mu = x^T x$$

$$\gamma = x^T y$$

Which of the two inner products would you expect to be less sensitive to the perturbations caused by the finite precision of IEEE floating point arithmetic? Justify your answer.

Problem 1.5

Consider the function

$$f(x) = \frac{1.01 + x}{1.01 - x}$$

1.5.a. Find the absolute condition number for $f(x)$.

1.5.b. Find the relative condition number for $f(x)$.

1.5.c. Evaluate the condition numbers around $x = 1$.

1.5.d. Check the predictions of the condition numbers by examining the relative error and the absolute error

$$err_{rel} = \frac{|f(x_1) - f(x_0)|}{|f(x_0)|}$$
$$err_{abs} = |f(x_1) - f(x_0)|$$

with $x_0 = 1$, $x_1 = x_0(1 + \delta)$ and δ small.