Homework 5 Introduction to Computational Finance Spring 2023

Solutions due Friday April 14, 2023

Answers to the homework problems and programming tasks should be submitted using the class canvas page.

You should submit pdf files. **Do not sent Word files or any other text processing tool's input file**.

As with all homework assignments you are allowed and encouraged to consult the relevant literature. You are also expected **to cite all literature that is used to generate your solutions and your solutions must make clear your understanding of the work cited.**

Programming Assignment

Problem 5.1

Codes

1. Write a quadrature algorithm codes using the composite midpoint rule and the composite Trapezoidal rule. Both should use global step adaptation to refine the entire interval of integration to achieve a prescribed accuracy, i.e., when the estimated error is not small enough create a fine grid with a new step size αh with $0 < \alpha < 1$. Your choice of α should be such that there is complete reuse of the function evaluations from the earlier grids that is, $\alpha = 1/2$ for the composite Trapezoidal rule and $\alpha = 1/3$ for the composite midpoint rule. Your codes should exploit the complete reuse of coarse mesh function evaluations and write the fine mesh quadrature result in terms of an update to the coarse mesh result and the new function values.

2. The interface for the function should be something like (F, a, b, tol) where F is the function to be integrated, $[a, b]$ is the interval of integration, and tol is the absolute tolerance. Test your routine on some integrals for which you have an analytical solution that demonstrates the behavior of the method, e.g., convergence, accuracy etc. As always create a structured argument that your code is correct. This, of course, must make clear which method you used and your analysis and experimental design must relate to the method used.

Tasks

1. All of the following have symbolic solutions that are easily derived and these should be used in your work to assess true error, predict expected behavior and analyze observed

behavior. (You should verify the solutions to the integrals before using them.)

$$\int_0^3 e^x dx = e^3 - 1 \tag{1}$$

$$\int_0^{\frac{\pi}{3}} e^{\sin(2x)} \cos(2x) dx = \frac{1}{2}\left(-1 + e^{\frac{\sqrt{3}}{2}}\right) \tag{2}$$

$$\int_{-2}^1 \tanh(x) dx = \ln\left(\frac{\cosh(1)}{\cosh(2)}\right) \tag{3}$$

$$\int_0^{3.5} x\cos(2\pi x) dx = -\frac{1}{2\pi^2} \tag{4}$$

$$\int_{0.1}^{2.5} \left(x + \frac{1}{x}\right) dx = \frac{2.5^2 - 0.1^2}{2} + \ln(2.5/0.1) \tag{5}$$

2. **Make sure to describe carefully how you estimate the error and update the mesh.** You should compare your final mesh to that predicted by an apriori error estimation and to the true error on test problems for which you know the true solution.

3. Compare and comment on the accuracy of the composite midpoint rule and the composite trapezoidal rule for these problems.

4. The cumulative distribution function for a Gaussian random variable with mean 0 and variance 1 is

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-0.5t^2} dt$$

   and has no analytical form that can be used to define a computational algorithm. Its evaluation therefore requires estimating the improper integral above. Two techniques have been discussed in the class notes. Use either the composite midpoint rule or the composite Trapezoidal rule along with one of the improper integral techniques in the notes to find $P(x)$ for the value $x = 1.0$ with accuracy to at least 6 digits. Provide an analysis that shows you have evaluated all of the required values to sufficient accuracy to satisfy this requirement.

Written Exercises

There are no written problems in this assignment.