Homework 4 Foundations of Computational Mathematics 2 Fall 2019

Problem 4.1

Let $T \in \mathbb{R}^{n\times n}$ be a symmetric tridiagonal matrix, i.e., $e_i^T T e_j = e_j^T T e_i$ and $e_i^T T e_j = 0$ if $j < i-1$ or $j > i+1$. Consider $T = QR$ where $R \in \mathbb{R}^{n\times n}$ is an upper triangular matrix and $Q \in \mathbb{R}^{n\times n}$ is an orthogonal matrix.

Recall, the nonzero structure of R was derived in class and shown to be $e_i^T R e_j = 0$ if $j < i$ (upper triangular assumption) or if $j > i+2$, i.e, nonzeros are restricted to the main diagonal and the first two superdiagonals.

(4.1.a) Show that Q has nonzero structure such that $e_i^T Q e_j = 0$ if $j < i-1$, i.e., Q is upper Hessenberg.

(4.1.b) Show that $T_+ = RQ$ is a symmetric triagonal matrix.

(4.1.c) Prove the Lemma in the class notes that states that choosing the shift $\mu = \lambda$, where λ is an eigenvalue of T, results in a reduced T_+ with known eigenvector and eigenvalue.

Problem 4.2

(Golub and Van Loan Problem 8.3.1. (3rd Ed.) p. 423, Golub and Van Loan Problem 8.3.1. (4th Ed.) p. 465)

Suppose λ is an eigenvalue of a symmetric tridiagonal matrix T. Show that if λ has algebraic mulitplicity k, then at least $k-1$ of T's subdiagonal elements are zero.

Problem 4.3

(Golub and Van Loan Problem 8.3.6. (3rd Ed.) p. 424, Golub and Van Loan Problem 8.3.5. (4th Ed.) p. 465)

Show that if $A = B + iC$ is Hermitian then,

$$M = \begin{pmatrix} B & -C \\ C & B \end{pmatrix}$$

is symmetric. Relate the eigenvalues and eigenvectors of A and M.

Problem 4.4

(Golub and Van Loan Problem 8.3.8. (3rd Ed.) p. 424, Golub and Van Loan Problem 8.3.7. (4th Ed.) p. 465)

Suppose $A = S + \sigma uu^T$ where S is skew symmetric ($-S = S^T$), $u \in \mathbb{R}^n$ has unit 2-norm, and $\sigma \in \mathbb{R}$. Show how to compute an orthogonal Q such that $Q^T AQ$ is tridiagonal and $Q^T u = e_1$.