Graded Homework 2 Foundations of Computational Math 2 Spring 2022

The solutions must be submitted using the Canvas course page by 11:59PM on Sunday February 20, 2022.

Programming Exercise

Codes

1. **Piecewise interpolating polynomial**: A routine for the piecewise interpolating polynomial $g_d(x)$, on $[a, b]$ where the degree d may be $1, 2$, or 3 using a generic implementation of $p_{d,s}(x)$ per interval $I_s = [x_{sd}, x_{sd+d}]$.

2. **Piecewise interpolating polynomial**: A routine for the piecewise interpolating polynomial $g_d(x)$, on $[a, b]$ where the degree d may be $1, 2$, or 3 using a reference interval implementation of $p(z)$ based on Chebyshev points of the second kind and the Barycentric 2 Lagrange form.

3. **Piecewise cubic Hermite interpolating polynomial:** A routine for the piecewise interpolating polynomial $H_3(x)$, on $[a, b]$ where on each interval $I_i = [x_i, x_{i+1}]$ $H_3(x)$ is defined by the cubic polynomial, $p_{3,i}(x)$, that interpolates (x_i, f_i), (x_i, f_i'), (x_{i+1}, f_{i+1}), (x_{i+1}, f_{i+1}').

4. Implement the supporting code required to empirically validate the correct functioning of your codes and the tasks below.

Code Comments:

1. Your codes should be able to run in single or double precision (assumed to be IEEE standard FP).

2. Your codes must be efficient in time and space and make sure you discuss these aspects of your implementations. Include a discussion of how you represent and evaluate the local polynomials defining the piecewise functions.

Tasks

Task 1

Empirically validate the correct functioning of your codes. You must design experiments and describe the outcomes that provide evidence that your codes are working.

Task 2

Empirically compare the convergence of the three types of piecewise interpolating functions you have implmented in terms of a discrete approximation of the infinity norm of the error. Identify any differences you see in convergence performance based things such as local mesh choice, maximum interval size H_{max}, the use of Hermite interpolation etc.

Task 3

Empirically compare the convergence you observe for Task 2 to the convergence observed when you use a single interpolating polynomial of increasing degree based on Chebyshev points of the first or second kind. You should have these codes from your solutions to the first programming homework.

Task 4

Use the error bounds for interpolation to derive H_{max} to guarantee a particular accuracy for a given function and piecewise interpolating polynomial. Empirically evaluate whether or not your H_{max} value set this way is conservative or not.