You may use a calculator. Please, show all your work and circle you answers. State R commands you used.

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<th>Questions</th>
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<th>Actual</th>
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1) You have daily 30-year mortgage rate and 10-year Treasury yield data (the first column is the mortgage rate, the second is the 10-year Treasury yields in percent):
   read.table(url("http://www.math.fsu.edu/~goncharo/MIA/mortgage.txt"))
   The 10-year Treasury yield is a predictor variable.
   a) Estimate the change in the mean mortgage rate with a 95% confidence interval when Treasury yield increases by 1 (percent).

b) Conduct a \( t \)-test (with 95% confidence) to determine if the slope coefficient in the linear regression function is less than 1. Find the \( p \)-value.

c) Show graphically the meaning of the power of your test in part b) if “in fact” the increase of the 10-year Treasury yield gives the same increase in the mortgage rate (graph distributions, “hypothesizes”, etc. “Indicate” the power). Compute the power. Comment.
2) Refer to problem 3):
   a) Obtain a 95% confidence interval for the mean mortgage rate when the Treasury is 4.4%.

   b) Obtain a 95% prediction interval for the mortgage rate when the Treasury is 4.4%. Is this interval wider, narrower or the same as in part a)? Why?

   c) Obtain a Benferroni joint 95% confidence interval for the mortgage rate when the Treasury is 4.4% and 4.6%.
3) A set of random numbers was generated with the distribution given by the density function below. How the qqplot of these random numbers might look like? Graph it!

![Graph of probability density function and QQ-plot]

4) Get the ordinary (not weighted) least square estimation for the parameter $\beta$ in the following regression model: $Y_i = \beta X_i^{1/2} + \varepsilon_i$, where $\varepsilon_i$ are independent $N(0, X_i)$ $i=1, \ldots, n$

State the criterion and show all the steps.
5) Estimate the parameter $b$ in the linear regression model above (problem 4) with the help of method of maximum likelihood.
   a) What is the distribution of $Y_i$? What is the probability density function of this random variable?

   b) What is the likelihood function for one observation $(X_i, Y_i)$?

   c) What is the likelihood function for $n$ observations $\{(X_i, Y_i)\}_{i=1}^n$?

   d) What is the MML estimate of $\beta$?