Chapter 5: Angles and Arcs

5.1: Angles: Set of points determined by two rays.

Terminal side

A

O

B

α

Vertex

Initial side

Positive

Negative

Notes:
1) One unit of measurement for angles is the degree.
2) Angle in standard position obtained by one complete revolution in the counterclockwise direction has measure 360 degree (written 360°)
3) A right angle is a 90° and equal to \( \frac{1}{4} \) revolution
4) A straight angle is a 180° and equal to \( \frac{1}{2} \) revolution
5) Quadrantal angles: 90°, 180°, 270°, …
6) \( 1° = \frac{1}{360} \) revolution

EX: Draw each angle
a) 60°, b) −45°, c) 225°, d) −210°, e) 405°

Radian: A central angle has a measure 1 radian if it intercepts an arc with length equal to the radius of the circle.

Notes:
1) \( 2\pi \) radian = 1 revolution = 360°
2) \( \pi \) radian = 180°
3) 1 radian = \(\left(\frac{180}{\pi}\right)^\circ \approx 57.2958^\circ\)

4) \(1^\circ = \frac{\pi}{180} \text{ radian} \approx 0.0174533 \text{ radian}\)

Notes:

1) To change radians to degree, multiply by \(\frac{180}{\pi}\)

2) To change degrees to radian, multiply by \(\frac{\pi}{180}\)

EX: Convert the following angles to degree measures
   a) \(-\frac{\pi}{3} \text{ rad.}\)  b) \(\frac{3\pi}{4} \text{ rad.}\)  c) \(-\frac{5\pi}{6} \text{ rad.}\)

EX: Convert the following angles to radian measures
   a) \(210^\circ\)  b) \(-405^\circ\)

**Arc Length Formula:**

If an arc of length \(S\) on a circle of radius \(r\) subtends a central angle of radian measure \(\theta\) then

\[ S = r\theta \]

EX: **Arc Length:**

1) A central angle \(\theta\) is subtended by an arc 10 cm long on a circle of diameter 8 cm. Find the measure of \(\theta\) in  a) radian  b) degree

2) Find \(S\) given  \(r = 3\text{ ft}, \theta = \frac{7\pi}{2}\)

3) Find \(S\) given  \(r = 5\text{ ft}, \theta = 144^\circ\)

4) Find the number of radians in \(\frac{3}{8}\) revolution

5) A bike has wheels that are 28 inches in diameter. How far does the bike move as wheels roll through an angle of \(15^\circ\)

**Area of a Circular Sector:**

\[ A = \frac{1}{2} r^2 \theta \]

\(r = \text{radius}\)
\(\theta = \text{central angle measures in radians}\).
**EX: Area of a circular Sector:**

1) If \( \theta = 50^\circ \), \( r = 8 \text{ m} \) Find a) \( S \) b) \( A \)

2) Find the area of a sector of a circle of diameter 8 ft formed by an angle of 30\(^\circ\)

3) Find the area of a circular sector with central angle 1/4 revolution if the length of intercepted arc is \( \frac{4\pi}{3} \) centimeters.

3) The area of a sector of a circle with radius 3 centimeters is \( \frac{3\pi^2}{4} \) square centimeters. Find the length of

The intercepted arc in centimeters.

**Circular Motion:**

*If an object moves in a circular path, two speeds are involved.*

1) The rate at which distance is traveled along the circle, called linear speed \( v \)
2) The rate at which the object revolves about the center of the circle, called angular speed \( w \)

\[
\begin{align*}
    v & = \text{(length/time)} \\
    w & = \text{(radian/time)}
\end{align*}
\]

Also \( v = \)

**Notes:**

1) The time units in \( v \) and \( w \) must be the same
2) The linear units used in \( v \) and \( r \) must be the same

**Ex:**

1) A wheel is rotating at 200 revolutions per minute. Find the angular speed in radians per minute
2) An object is traveling around a circle with a radius of 2 m. If in 20 seconds the object travels 5 m, what is its angular speed? what is its linear speed?
3) A rock is spinning at 180 rpm at the end of a 2-foot rope. Find the rock’s linear speed (in ft/min).
4) The windshield wiper of a car is 18 inches long. How many inches will the tip of the wiper move during \( \frac{1}{3} \) revolution.

5) A pendulum swings through an angle of 15\(^\circ\) each second. If the pendulum is 20 inches long, how many inches does its tip move each second?
6) Find the radius (in feet) of a circle, if a central angle of 95\(^\circ\) subtends a 95 foot arc.