Section 7.2: The Law of Sines

If none of the angles of a triangle is right angle. The triangle is called **Oblique**.

- Three acute angles
- Two acute and one obtuse

**To Solve Oblique Triangle:**

**Case I:**

SAA or ASA

**Case II:**

SSA (angle opposite to one of the sides)

Use law of sine

**Law Of Sines:**
\[
\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}
\]

**Ex:** In the triangle ABC, if \( \alpha = 40^\circ \), \( \beta = 60^\circ \), \( a = 4 \). Find \( b, c \)?

**Ex:** In the triangle ABC, if \( \sin \beta = \frac{3}{4} \), \( b = 3 \), \( a = 2 \). Find \( \sin \alpha \)?

**The Ambiguous Case:**

**I)** If \( x < y \)

1) \( y \sin \theta < x \) \( \Rightarrow \) 2 triangles
2) \( y \sin \theta = x \) \( \Rightarrow \) 1 triangle (right)
3) \( y \sin \theta > x \) \( \Rightarrow \) No triangle

**II)** If \( x \geq y \) \( \Rightarrow \) 1 triangle

**EX:** How many triangles ABC can be constructed

1) with \( b = 2\sqrt{2} \), \( c = 4 \), \( \beta = 45^\circ \) ?
2) with \( a = 1 \), \( b = \sqrt{3} \), \( \alpha = 30^\circ \) ?
3) with \( a = \sqrt{3} \), \( c = 1 \), \( \gamma = 60^\circ \) ?
4) with \( a = 3 \), \( b = 2 \), \( \alpha = 140^\circ \) ?

**Note:** In navigation and surveying, the direction or bearing from a point O to a point P equal to the acute angle \( \theta \) between the ray OP and the Vertical line through O, the North-South line
1- A point $P$ on the level ground is 3 kilometers due north of a point $Q$. A runner proceeds in the direction $N 25^\circ E$ from $Q$ to a point $R$, then from $R$ to $P$ in the direction $S 70^\circ W$. Find the distance run.

2- Consult the figure. To find the length of the span of a proposed ski lift from $A$ to $B$ to, a surveyor measures the angle $DAB$ to be $25^\circ$ and then walks off a distance of 1000 feet to $C$ and measures the angle $ACB$ to be $15^\circ$. What is the distance from $A$ to $B$?

3- The angle of elevation of an airplane observed by two observers from two points $A$ and $B$ on level ground are $40^\circ$ and $35^\circ$ respectively. Point $A$ and $B$ are 1000 ft apart and the airplane is between the points, in the same vertical plane. a- How high is the airplane? b- Find the distance between the airplane and the observer at point $A$.

4- The angle of depression from a balloon to two points $A$ and $B$ on level ground are $52^\circ$ and $28^\circ$ respectively. Points $A$ and $B$ are 14 miles apart and the balloon is between the points, in the same vertical plane. Find the distance in miles between the balloon and the point $A$. 