3.7- Marginal Analysis in Business and Economics:

Marginal Cost, Revenue, and Profit.

If x is the number of units of a product produced in some time interval, then

Total Cost = \( C(x) \)
Marginal Cost = \( C'(x) \)

Total Revenue = \( R(x) \)
Marginal Revenue = \( R'(x) \)

Total Profit = \( P(x) = R(x) - C(x) \)
Marginal Profit = \( P'(x) \)

Note: \( R = xp \)

Where \( x \) = number of units
\( p \) = price per unit

Ex. 1) The price demand equation and the cost function for the production of television sets are given, respectively by:
\( x = 9,000 - 30p \) and \( C(x) = 150,000 + 30x \)

Where \( x \) is the number of sets that can be sold at a price of \( $p \) per set and \( C(x) \) is the total Cost (in dollars) of producing \( x \) sets.

a) Express the price \( p \) as a function of \( x \)
b) Find the marginal cost.
c) Find the Revenue function in terms of \( x \)
d) Find the marginal Revenue
e) Find \( R'(3,000) \) and \( R'(6,000) \), and interpret.
f) Graph the Cost function and the Revenue function on the same coordinate system for \( 0 \leq x \leq 9,000 \). Find the break-even points and indicate regions of loss and profit.
g) Find the profit function in terms of \( x \).
h) Find the marginal profit.
i) Find \( P'(1,500) \) and \( P'(4,500) \), and interpret.
Marginal Average Cost, Revenue, and Profit:

If \( x \) is the number of units of a product produced in some time interval, then

\[
\text{Average Cost} = \overline{C}(x) = \frac{C(x)}{x}, \quad \text{cost per unit}
\]

Marginal Average Cost = \( \overline{C}'(x) \)

\[
\text{Average Revenue} = \overline{R}(x) = \frac{R(x)}{x}, \quad \text{revenue per unit}
\]

Marginal Average Revenue = \( \overline{R}'(x) \)

\[
\text{Average Profit} = \overline{P}(x) = \frac{P(x)}{x}, \quad \text{profit per unit}
\]

Marginal Average Profit = \( \overline{P}'(x) \)

Ex. 2) The total cost (in dollars) of printing \( x \) dictionaries is

\[C(x) = 20,000 + 10x\]

a) Find the average cost per unit if 1,000 dictionaries are produced.
b) Find the marginal average cost at a production level of 1,000 units and interpret.
c) Estimate the average cost per dictionary if 1,001 dictionaries are produced.

Ex. 3) The total profit (in dollars) from the sale of \( x \) portable stereos is

\[P(x) = 22x - \frac{x^2}{10} - 400\]

a) find the exact profit from the sale of the 41\(^{st}\) stereo and interpret
b) Use the marginal profit to approximate the profit from the sale of the 41\(^{st}\) stereo and interpret.

Ex. 4) The total cost (in dollars) of producing \( x \) electric guitars is

\[C(x) = 1,000 + 100x - \frac{x^2}{4}\]

a) Find the exact cost of producing the 51\(^{st}\) guitar.
b) Use the marginal cost to approximate the cost of producing the 51\(^{st}\) guitar and interpret.