### 4.2 First Derivative and graphs

**Increasing (↑) and decreasing (↓) functions**

![Graph of a function](image)

- $m > 0$
- $m = 0$
- $m < 0$

**In general**

- If $f'(x) > 0$ on $(a, b) \rightarrow f(x)$ is $\uparrow$
- If $f'(x) < 0$ on $(a, b) \rightarrow f(x)$ is $\downarrow$
Steps for finding where $f(x)$ is $\uparrow$ or $\downarrow$

1. Find $f'(x)$
2. Set $f'(x) = 0$ and solve for $x$
3. Construct a sign chart for $f'(x)$ using $x$ values in step 2
4. Pick test values around the values of $x$ found in step 2, and substitute those values into $f'(x)$ to determine where $f'(x) > 0$ and $f'(x) < 0$

Example: Find the intervals where $f(x)$ is $\uparrow$ or $\downarrow$

1. $f(x) = 8x - x^2$
2. $f(x) = 1 + x$
3. $f(x) = (1-x)^{\frac{1}{3}}$
4. $f(x) = \frac{1}{x-2}$
Critical values of \( f \) (c.v.)

The values of \( x \) in the domain of \( f \) where \( f'(x) = 0 \) or \( f'(x) \) DNE are called the critical values of \( f \).

Note: The c.v. of \( f \) are always partition numbers for \( f' \), but \( f' \) may have partition numbers that are not c.v.'s.

Local Extrema (min. & max.)
Thm 1: Existence of local extrema

If \( f \) is cont. on the interval \((a, b)\) and \( f(c) \) is a local extremum, then either \( f'(c) = 0 \) or \( f'(c) \) DNE.

1st derivative test \([f' \text{ chart}]\)

Let \( c \) be a c.v. of \( f \)

\( f(c) \) defined \( f'(c) = 0 \) \( f'(c) \) DNE

Use \( f' \) chart

If \( f \) is \( \nearrow \) then \( \nearrow \) \( f(c) \) is local max

If \( f \) is \( \searrow \) then \( \searrow \) \( f(c) \) is local min

If \( f \) is \( \nearrow \) \( \searrow \) \( f(c) \) is not local extremum
ex Given \( f(x) = x^3 - 9x^2 + 24x - 10 \)
find local extrema and graph

HW Graph \( f(x) = x - x^3 \)

ex If \( f \) is cont. on \( (-\infty, \infty) \)
and
\[
\begin{array}{c|cccc}
& -1 & 0 & 2 & \\
f' & + & ND & - & \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
\hline
x & -2 & -1 & 0 & 1 & 2 & 4 & \hline
y & 4 & 2 & 1 & 2 & 1 & 0 & \\
\hline
\end{array}
\]

Graph \( f \)

\[
\frac{41}{256}
\]

If \( f(-2) = 4, \ f(0) = 0, \ f(2) = -4 \)
\[
f'(-2) = 0, \ f'(0) = 0, \ f'(2) = 0
\]
\[
f'(x) > 0 \text{ on } (-\infty, -2) \text{ and } (2, \infty)
\]
\[
f'(x) < 0 \text{ on } (-2, 0) \text{ and } (0, 2)
\]

Graph \( f \)
Given \( y = f'(x) \), find where \( f' \) is positive and negative, local min and max, then sketch possible graph of \( y = f(x) \), \( y = f'(x) \).