5.2: Trig. Functions: Unit Circle approach:

If $r \neq 1$, find the values for all the Trig functions.

Let $p = (a, b)$ a point on the terminal side of $\theta$ that intersects the circle.

$$a^2 + b^2 = r^2$$

1) sine function: $\sin \theta = \frac{b}{r}$
2) cosine function: $\cos \theta = \frac{a}{r}$
3) tangent function: $\tan \theta = \frac{b}{a}$
4) cotangent function: $\cot \theta = \frac{a}{b}$
5) secant function: $\sec \theta = \frac{r}{a}$
6) cosecant function: $\csc \theta = \frac{r}{b}$

Ex: If $(-5, 12)$ is a point on the terminal side of an angle $\theta$, find the exact value for the remaining Trig. Functions.

Note: If $r = 1$, then we have a unit circle. We conclude that $a^2 + b^2 = 1$ and

$$\sin \theta = \frac{b}{r}, \quad \cos \theta = \frac{a}{r}, \quad \tan \theta = \frac{b}{a}, \quad \cot \theta = \frac{a}{b}, \quad \sec \theta = \frac{r}{a}, \quad \csc \theta = \frac{r}{b}$$

Ex: 1) If $p(x, -\frac{\sqrt{2}}{3})$ is on the unit circle such that $x > 0$, find $\cot \theta$ where $p$ is on the terminal side of the angle of $\theta$ radians.

Finding the exact values of the six Trig. Functions of quadrant angles.
Ex: Find the coordinate of \( p(x, y) \) on the unit circle and on the terminal side of the angle \( \frac{7\pi}{2} \).

Finding Exact values Trig functions of special angles:

1) \( \theta = \frac{\pi}{4} \) or \( 45^\circ \)

2) \( \theta = \frac{\pi}{6} \) or \( 30^\circ \)

3) \( \theta = \frac{\pi}{3} \) or \( 60^\circ \)
Ex: Find the exact value of each expression.

1) \( \sqrt{3} \tan \frac{\pi}{3} + 2 \cos \frac{\pi}{3} \), 2) \( \sec \pi - \csc \frac{\pi}{2} \), 3) \( \sin \frac{3\pi}{2} + \tan \pi \), 4) \( \csc 60^\circ + \cot 45^\circ \)
5) \( \sec 30^\circ \sin 45^\circ \)

Notes:

1) Odd and even Trig. Functions

2) Trig. Function signs.

REducing Functions of an Angle in Any Quadrant

1) \( \sin(\pi - \theta) = \sin(\pi + \theta) \)
2) \( \cos(\pi - \theta) = \cos(\pi + \theta) \)
3) \( \tan(\pi - \theta) = \tan(\pi + \theta) \)
4) \( \sin(2\pi - \theta) = \sin(2\pi + \theta) \)
5) \( \cos(2\pi - \theta) = \cos(2\pi + \theta) \)
6) \( \tan(2\pi - \theta) = \tan(2\pi + \theta) \)

Note: 1) If \( \theta \) is an acute angle, then any function of \((2\pi - \theta)\), \((2\pi - \theta)\) or \((\theta - \pi)\) is reducible to the same-named function of \(\theta\),

7) \( \sin(\frac{\pi}{2} - \theta) = \sin(\frac{\pi}{2} + \theta) \)
8) \( \cos(\frac{\pi}{2} - \theta) = \cos(\frac{\pi}{2} + \theta) \)
9) \( \tan(\frac{\pi}{2} - \theta) = \tan(\frac{\pi}{2} + \theta) \)
10) \( \sin(\frac{3\pi}{2} - \theta) = \sin(\frac{3\pi}{2} + \theta) \)
11) \( \cos(\frac{3\pi}{2} - \theta) = \cos(\frac{3\pi}{2} + \theta) \)
12) \( \tan(\frac{3\pi}{2} - \theta) = \tan(\frac{3\pi}{2} + \theta) \)

Note: 2) whereas any function of \((\pi/2 - \theta),(\pi/2 + \theta),(3\pi/2 - \theta)\) or \((3\pi/2 + \theta)\) is reducible to the complementary-named function of \(\theta\)

Ex: Evaluate

1) \( \sin 150^\circ \), 2) \( \cos(-\frac{\pi}{3}) \), 3) \( \sec 210^\circ \), 4) \( \tan 300^\circ \), 5) \( \sin(-\pi) \), 6) \( \cos \frac{7\pi}{6} \)

Ex: Find the exact value for \( \tan 60^\circ + \tan 150^\circ \)
Ex: If \( \cos \theta = -0.3 \) find

\[
\cos(-\theta) + \cos(\pi - \theta) + \cos(5\pi + \theta) - \cos(6\pi - \theta)
\]

Ex: Using odd and even function definition evaluate the following.
1) \( \sin(\theta - \pi) = \)
2) \( \cos(\theta - \pi) = \)
3) \( \tan(\theta - 2\pi) = \)

Ex: Find the x coordinate of \( p(x, y) \) on the unit circle and on the terminal side of the angle \(-\frac{7\pi}{6}\)

Ex: If \( \cot \theta = -\frac{1}{\sqrt{3}} \), find all possible values for \( \theta \) for \( 0 \leq \theta < 2\pi \)

Ex: Suppose that the terminal point determined by \( \theta \), is the point \( \left(\frac{7}{25}, \frac{24}{25}\right) \) on the unit circle. Find the terminal point determined by \( \theta - 3\pi \).

Ex: Suppose that the terminal point determined by \( \theta \), is the point \( \left(\frac{7}{25}, \frac{24}{25}\right) \) on the unit circle. Find the terminal point determined by \( -\theta - 4\pi \).

Ex: Suppose that the terminal point determined by \( \pi - \theta \), is the point \( \left(-\frac{7}{25}, \frac{24}{25}\right) \) on the unit circle. Find the terminal point determined by \( 8\pi - \theta \).

Ex: If \( \theta \) is an acute angle and the terminal side is determined by \( \theta \), then the terminal side determined by \( 5\pi + \theta \) will be in quadrant.

Ex: If \( \theta \) is an acute angle and the terminal side is determined by \( \theta \), then the terminal side determined by \( \theta - \frac{11\pi}{2} \) will be in quadrant.

Algebra question: Simplify

\[ \frac{-\sqrt{2}}{2} + 1 \]