8.3 Maxima and Minima

Thm 2: Let $f(a,b)$ be a local extremum for the function $f$. If both $f_x$ and $f_y$ exist at $(a,b)$, then $f_x(a,b) = 0$ and $f_y(a,b) = 0$

$(a,b)$ called critical point

Ex: Given $f(x,y) = -x^2 + 2xy - 2y^2 - 4x + 12y - 5$

Find the critical value(s) of $f$
Thm 2: 2nd derivative test for local extrema

Given

1. \( z = f(x, y) \)
2. \( f_x(a, b) = 0 \) and \( f_y(a, b) = 0 \)
3. all 2nd order partial derivatives of \( f \) exists
4. \( A = f_{xx}(a, b), \quad B = f_{xy}(a, b), \quad C = f_{yy}(a, b) \)

Then

1. If \( AC - B^2 > 0 \) and \( A < 0 \) \( \Rightarrow f(a, b) \) is local max
2. If \( AC - B^2 > 0 \) and \( A > 0 \) \( \Rightarrow \) local min
3. If \( AC - B^2 < 0 \) \( \Rightarrow f \) has a saddle pt at \( (a, b) \)
4. If \( AC - B^2 = 0 \), the test fails
Ex Use Thm 2 to find local extrema for 
\[ f(x,y) = x^3 + y^2 - 6xy \]

#36) A rectangular box with no top and two intersecting partitions is to be made to hold a volume of 72 in\(^3\). What should its dimensions be in order to use the least amount of material in its construction?

Ex A rectangular box with eight compartments is to have a volume of 30 in\(^3\). Find the dimensions that will require the least amount of material.
Ex. A store sells two brands of certain item each month. If the total profit (in hundreds of dollars) is given approximately by \( P(x,y) = 15 + 16x + 12y - 4x^2 - 2y^2 \), where \( x \) is the number of units of Brand A and \( y \) is number of units of Brand B. Find the maximum profit possible each month from the sale of these two items.

Ex. The cost function \( C \) (in hundreds of dollars) of producing two products is

\[ C(x,y) = 3x^2 + y^2 + 3xy - 3y - 6x + 15 \]

where \( x \) is the quantity of product A and \( y \) is the quantity of product B. Find the minimum cost of producing these products.