MGF1106 Lecture Outlines – Ms. Harris Sp'2017

Part 1 Module 1 - Sets, elements, subsets

Any collection of objects is a _______________.
The objects contained in a set are called its _______________ or ____________________.
The ______________________defines a particular set by listing its elements, separated by commas, within curly brackets.

Names for sets

It is conventional to use capital letters for names of sets.

S = {a, b, c}   A = {1, 2, 3, 4, 5}   T = {Moe, Larry, Curly}   B = {cat, dog}

Sets, elements

Referring to the sets   S = {a, b, c}   and   T = {Moe, Larry, Curly},
we observe, for example, that "a ____________________________ S" and "Larry ____________________ T."

These statements are denoted symbolically, as follows:

We can also observe that ___________________________________ and ____________________________________
These statements are denoted symbolically, as follows:

Cardinality

The ____________________________ of a set is the number of elements contained in that set.
For example, let S = {a, b, c} and B ={dog, cat}.
Then, the cardinality of S is _________________
and the cardinality of B is ___________________.
These facts are denoted symbolically, as follows:

In general, if V is any set, then ________________ represents the ______________________ in set V.

Set Equality

Two sets are equal if they ____ elements.
For example

{1, 2, 3, 4, 5} = {5, 3, 1, 4, 2}
{Moe, Larry} = {Moe, Larry}
{b, c, a} = S (referring to the set S defined earlier).

However,

{1, 2, 3, 4, 5}   {a, b, c, d, e}

More formally, if V and W are any sets, then **V = W**
means that every element of V is ______________________________W
and every element of W _______________________________V. Note that the order in which the elements are listed is not important.

Subsets (informal definition)

Suppose V is a set, and W is a set that is formed using only the elements of V
(that is, W can be formed entirely by including/excluding elements of V).
We say that W _______________________________________V.
This is conveyed symbolically as follows:

Example
Let T = {Moe, Larry, Curly}. List all the subsets of T.

{ } This set is known as the __________________, and it is also a subset of T.

Observe the following, from the previous example.

1. {Moe, Larry, Curly} {Moe, Larry, Curly}
   General fact: Every set is _______________________________________.
2. { } {Moe, Larry, Curly}

General fact: The empty set is a ________________________________.

Here is a more formal definition of the term **subset:**

Suppose V and W are sets, and that every element of W is also an element of V.
Then W V.
This is equivalent to saying that W is a subset of V, if there is no element of W that isn't also an element of V.

Examples (True or false)

1. {b, h, r, q} {h, r} A. True B. False
2. {3, 12, 5, 19} {19, 3, 5, 12, 18} A. True B. False
3. {a, 13, d, 2} {13, 2, d, a} A. True B. False

Proper Subsets

Suppose W is a subset of V, but W is not equal to V.
Then W is a ______________________________of V.
This is denoted

A proper subset is a subset that is not equal to the set it is a subset of.
A proper subset is a subset that is "smaller than" the set it is a subset of.
A moment ago, we listed all the subsets of the set {Moe, Larry, Curly}. Here they are:

1. { } 2. {Moe} 3. {Larry}
4. {Curly} 5. {Moe, Larry} 6. {Moe, Curly}
7. {Larry, Curly} 8. {Moe, Larry, Curly}

Which of these are also proper subsets of {Moe, Larry, Curly}?
Answer: ____

Complements and Universal Sets

Let T = {Moe, Larry, Curly}. List all elements that aren't in T.

Universal Sets

For a particular discussion or sequence of exercises in set mathematics, a Universal Set (U) is "larger" set that contains, ____.
A universal set serves as a frame of reference for the discussion.

Example
Let U = {Moe, Larry, Curly, Shemp, Curly Joe}.
Let T = {Moe, Larry, Curly}.
List a set containing all the elements that aren't in T.

We call this set the ______________________________, denoted

Set-builder notation

So far, we have been using the roster method for defining sets. The roster method defines a set by listing its elements. An alternative approach is to define a set by _______________ its elements, rather than literally listing them.
Set-builder notation is a way of defining a set by describing its elements, rather than listing them.

Example
Let U = {a, b, c, d, e, f}.
The universe for this discussion consists of six letters, four of which are consonants, two of which are vowels. Within this universe, let C be the set containing all the consonants, and let V be the set containing all the vowels.
C = {x|x is a consonant}
C is "______________________________, such that _________________________________."
C = {x|x is a consonant} = { }

V = {x|x is a vowel}
V is "______________________________, such that __________________________."
V = {x|x is a vowel} = { }

Exercises

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {3, 5}
E = {2, 4, 6, 8, 10}
F = {x|x is an odd number}

True or false:

1. E F	A. True	B. False
2. F A	A. True	B. False
3. {1, 2, 3} {2, 3, 5, 6}	A. True	B. False
4. {4, 7} {4, 7}	A. True	B. False
5. {4, 7} {4, 7}	A. True	B. False
6. {4} {4, 7}	A. True	B. False

Let U = {Moe, Larry, Curly, Shemp, Curly Joe}
T = {Moe, Larry, Curly}

True or False:

{ } { }	A. True	B. False

The number of subsets in a finite set

Common sense suggests that the more elements a set has, the more subsets it will have.
As we shall see, there is a precise relationship between the _________________________________and the _______________________________it has.

For a set V, suppose n(V) = k. Then V has ___________________ subsets and _____________________ proper subsets.

Part 1 Module 2 - Set operations, Venn diagrams

Set operations

Let U = {x|x is an English-language film}
Set A below contains the five best films according to the American Film Institute.
Set B below contains the five best films according to TV Guide.
Set C below contains the five most passionate films according to the American Film Institute.

A = {*Citizen Kane, Casablanca, The Godfather, Gone With the Wind, Lawrence of Arabia*}
B= {*Casablanca, The Godfather Part 2, The Wizard of Oz, Citizen Kane, To Kill A Mockingbird*}
C = {*Gone With the Wind, Casablanca, West Side Story, An Affair To Remember, Roman Holiday*}.
Form a new set whose elements are those that sets A and B have in common
A = {*Citizen Kane, Casablanca, The Godfather, Gone With the Wind, Lawrence of Arabia*}
B= {*Casablanca, The Godfather Part 2, The Wizard of Oz, Citizen Kane, To Kill A Mockingbird*}

This set is called the ______________________________ of A and B, denoted ________________________.
A ∩ B = ____.

Find B ∩ C

A different operation: form a new set that contains all the elements of A along with all the elements of B.
This set is called the ____, denoted ___________________.

We have encountered three basic set operations (including something from Part 1 Module 1).

Intersection	**S ∩ T =**	________________________________
Union	**S ∪ T =**	________________________________
Complement	**S′ =**	________________________________

Venn Diagrams

A Venn diagram is a drawing in which sets are _____________________________________
____.
Venn diagrams can be used to illustrate ________________________________, and the effects of set operations.
Venn diagrams are also used in other areas of mathematics, such as counting, probability and logic.

Intersection

Let S, T represent any sets in a universe U. The Venn diagram below illustrates the effect of __________________. The shaded region corresponds to __________________.

Union

Let S, T represent any sets in a universe U. The Venn diagram below illustrates the effect of __________________. The shaded region corresponds to __________________.

Complement

Let S, T represent any sets in a universe U. The Venn diagram below illustrates the effect of __________________. The shaded region corresponds to ___________.

Exercise #1

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

S = {3, 5, 8, 11}

T = {3, 4, 6, 7, 8, 10, 11}

V = {2, 5, 6, 7, 8}

W = {1, 3, 5, 6}

Find (V′ ∩ S′) ∩ (W ∪ T)

- A. { }
- B. {4, 10, 11}
- C. {1, 4, 10}
- D. None of these

Exercise #2

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

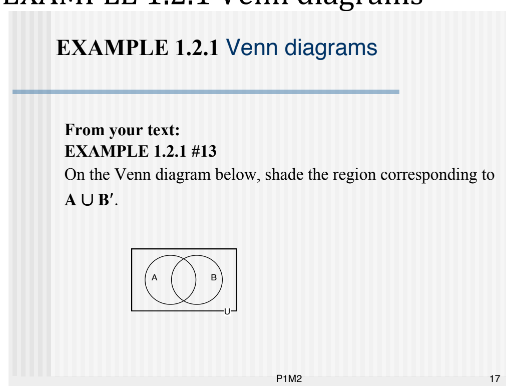
T = {3, 4, 6, 7, 8, 10, 11}

V = {2, 5, 6, 7, 8}

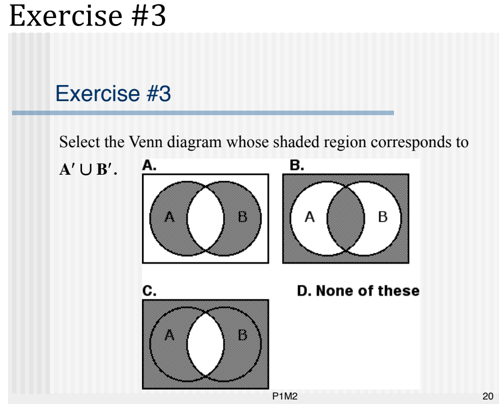
Find (V ∩ T′)′

- A. {1, 3, 4, 6, 7, 8, 9, 10, 11}
- B. {3, 4, 10, 11}
- C. {1, 3, 4, 6, 8, 9, 10, 11}
- D. None of these

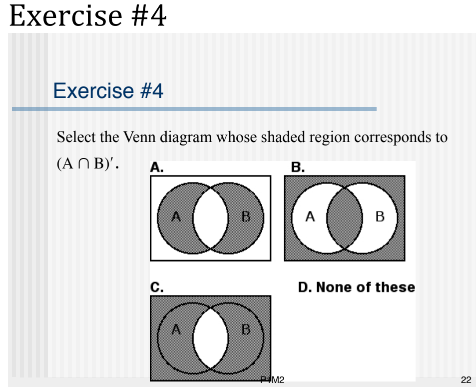
EXAMPLE 1.2.1 Venn diagrams



Exercise #3



Exercise #4



DeMorgan's Laws

In the previous two exercises we saw that the shaded figure for $(A \cap B)'$ is identical to the shaded figure for $A' \cup B'$. This means that $A' \cup B'$ and $(A \cap B)'$ are ______________________________.

This confirms one of the following general facts, known as **DeMorgan's Laws for Set Mathematics.**

For any sets S, T

$(S \cup T)' = S' \cap T'$

$(S \cap T)' = S' \cup T'$

"The ____________________________ is the intersection of the complements;
the ________________________________ is the union of the complements".

Exercise #5

Let U = {a, b, c, d, e, f, g}

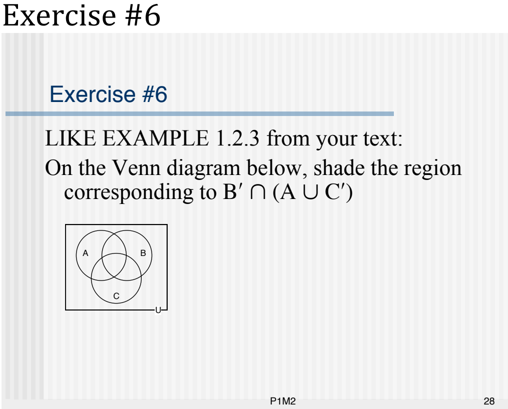
T = {c, e}

V = {a, d, e}

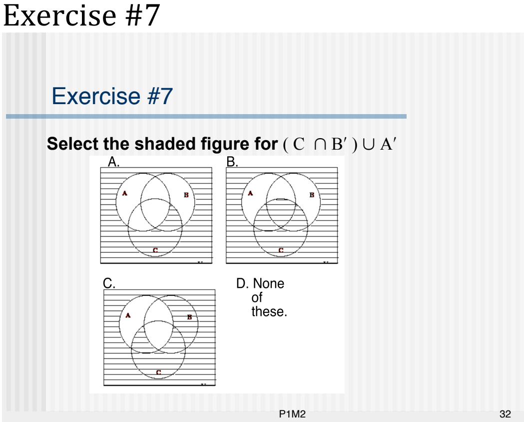
Find $(T' \cup V')'$

A. {e}
B. {a, c, d, e}
C. {b, f, g}
D. None of these

Exercise #6



Exercise #7



Part 1 Module 3 - Survey Problems and Venn diagrams

Exercise # 1 (example 1.3.1 from text)

A survey of 64 informed consumers revealed the following information:

45 believe that Elvis is still alive
49 believe that they have been abducted by space aliens
42 believe both of these things

1. How many believe Elvis is still alive or believe that they have been abducted by space aliens?
2. How many believe neither of these things?

We will use Venn diagram to organize this information.

Exercise #2

99 grumpy old people were surveyed regarding things they dislike.
66 dislike puppies 34 dislike kitties 32 dislike children
10 dislike puppies and dislike kitties and dislike children
28 dislike puppies and dislike children
8 dislike kitties and don't dislike children and don't dislike puppies
25 dislike puppies and dislike kitties

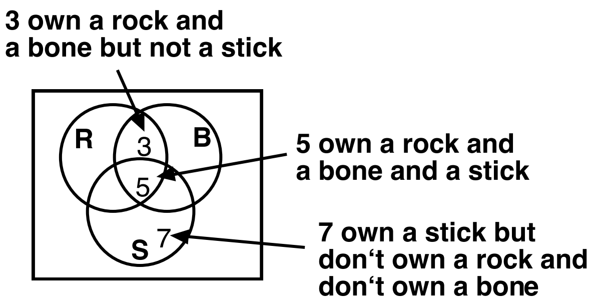
How many don't dislike puppies and don't dislike kitties and don't dislike children?

A. 50 B. 48 C. 16 D. 21 E. None of these

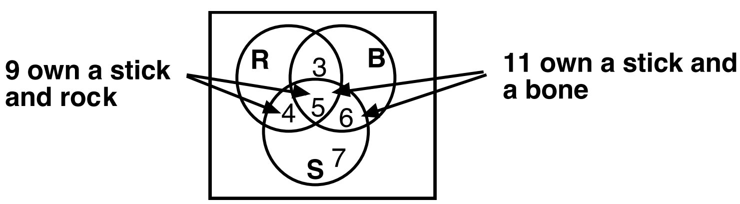
Guidelines for 3-circle survey problems

1. If a description names ____, then that description is pointing at ____________________________________ of the eight regions of the diagram, so the accompanying number ____________________________________.

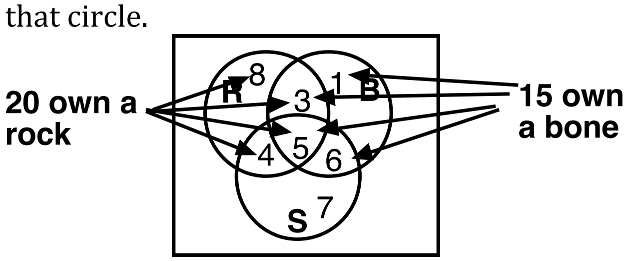
This is illustrated in the diagram below. Assume that a number of Neanderthals were surveyed regarding their favorite possessions. Set S represents those who own a stick, R represents those who own a rock, and B represents those who own a bone.



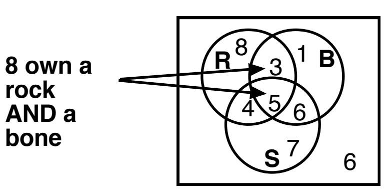
2. If a description names the ___________________________________, then that description is pointing at ____, so the accompanying number gets ______________________ regions of the diagram.

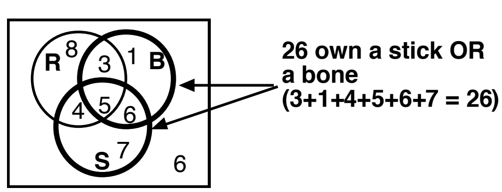


3. If a description ____________________________, then that description is pointing at ___________________________, so the accompanying number gets _________________________________ of that circle.



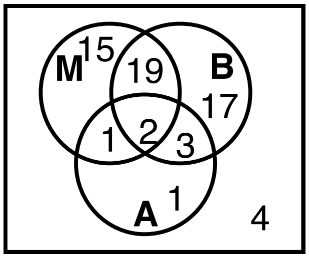
4. "And" means "intersection," or means "union."





Exercise #2
A number of young people were surveyed about their friends.
The results are summarized in the Venn diagram below.
How many have friends on MyFace.com and BookFace.com?

A. 19 B. 21 C. 57 D. None of these



M: Have –friends“ on MyFace.com

B: Have –friends“ on BookFace.com

A: Have actual, real friends

Exercise #3
Using the same Venn diagram as above, how many have exactly one of the three kinds of friends?

A. 23 B. 33 C. 2 D. None of these

Exercise #4
Using the same Venn diagram as above, how many have actual, real friends, but don't have friends on Bookface.com?

A. 1 B. 5 C. 2 D. None of these

Exercise #5
Using the same Venn diagram as above, how many have actual real friends, or have friends on MyFace.com?

A. 38 B. 16 C. 3 D. None of these

Part 1 Module 4 - The Fundamental Counting Principle

EXAMPLE 1.4.1
Plato is going to choose a three-course meal at his favorite restaurant. He must choose one item from each of the following three categories.

First course: Tofu Soup (TS); Seaweed Salad (SS)
Second course: Steamed Tofu (ST); Baked Tofu (BT); Fried Tofu (FT);
Third course: Tofu Cake (TC); Tofu Pie (TP); Seaweed Delight (SD)

How many different three-course meals are possible?

The Fundamental Counting Principle

We will list every possible 3-course meal:

1. TS-ST-TC	10. SS-ST-TC
2. TS-ST-TP	11. SS-ST-TP
3. TS-ST-SD	12. SS-ST-SD
4. TS-BT-TC	13. SS-BT-TC
5. TS-BT-TP	14. SS-BT-TP
6. TS-BT-SD	15. SS-BT-SD
7. TS-FT-TC	16. SS-FT-TC
8. TS-FT-TP	17. SS-FT-TP
9. TS-FT-SD	18. SS-FT-SD

There are 18 different 3 course meals.
You probably noticed that there is a more economical way to answer that question.
Choosing a three-course meal requires ______________________________:

1. Choose

2. Choose

3. Choose

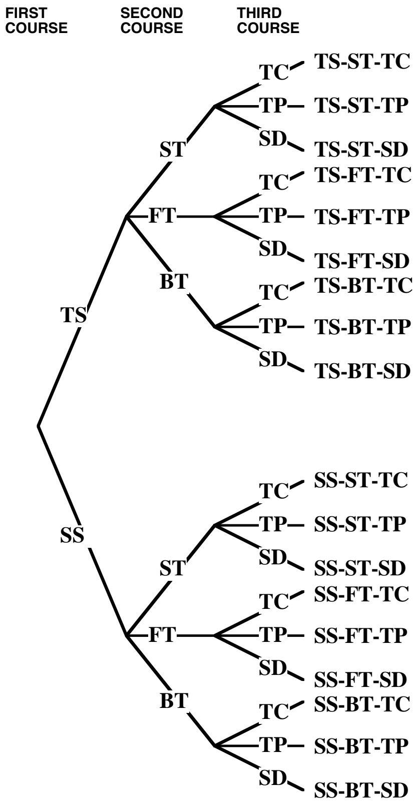
Then notice that 2 x 3 x 3 = 18

This illustrates the ______________________________, which describes a technique for determining the number of different outcomes in certain complex processes:

Step 1: Analytically break down the complex process into a number of __________________;
Step 2: Determine the ___________________________ for each decision identified in Step 1;
Step 3: ________________________ numbers from Step 2.

Why it works

The Fundamental Counting Principle works when we have an orderly decision process that has an ____. We are counting the number a branch-tips at the end of the tree.



The simplest Fundamental Counting Principle problems are those in which we are presented with a menu, and the situation dictates that we must choose one item from each category on the menu.

EXAMPLE 1.4.12
Gomer is considering the purchase of a new super-cheap sport/utility vehicle, the Skuzuzi Kamikaze. He must choose a vehicle, taking into account the following options:

i. Transmission: 4-speed standard transmission, 5-speed standard transmission, or automatic transmission;
ii. Bumper: steel bumpers, vinyl bumpers or 2x4 boards bolted to the front and back;
iii. Top: hard-top, vinyl top convertible, or chicken wire stapled over the roll bar;
iv. Funerary accessory: complementary funeral wreath or cremation urn.

How many different vehicle option packages are possible?

How many packages are possible if he already knows that he will order the chicken wire and can't order the steel bumpers?

Exercise – Prior to the start of a polo match, the captains of the two teams gather in the country club lounge. One team has 4 captains, and the other team has 3 captains.
If each captain of the first team shakes hands with each captain of the second team, how many handshakes occur?

Exercise - A quiz consists of 3 true/false questions followed by 4 multiple-choice questions, with 4 options for each multiple-choice question.
In how many ways is it possible to answer the 7 questions?

A. 264
B. 507
C. 2424
D. None of these.

EXAMPLE 1.4.6 - 1
There are 5 guys (including Gomer) on Gomer's bowling team. After the frame they will each choose one of the following: Dr. Pauper, Dr. Thunder, or Dr. Special.
How many outcomes are possible?

EXAMPLE 1.4.6 - 2
Again, there are 5 guys (including Gomer) on Gomer's bowling team. After the frame they will each choose one of the following: Dr. Pauper, Dr. Thunder, or Dr. Special.
However, Bill and Doug are having a spat, so they never agree on anything.
How many outcomes are possible, assuming that Bill and Doug will not order the same product?

A. 27 B. 108 C. 81 D. None of these

Exercise – 72 Dr. Pepper Clones
If each of the five guys in our previous example drank one of the 72 Dr. Pepper clones at the end of their bowling game, how many possible outcomes of choices are there?

Exercise
The dial on a combination lock has numbers ranging from 1 to 30. The "combination" that opens the lock is a sequence of three numbers. How many different combinations are possible, assuming that the combination may have repeated numbers, but the same number will not appear twice consecutively?

For example, 29-15-8, 7-13-22, 14-2-14, 8-29-15
are four different possible combinations, but 5-5-12 and 3-16-16 are not acceptable.

Example 1.4.11 #1
How many different 4-digit numbers can be formed
using the following digits? {0, 2, 3, 5, 8}
Note: the first digit cannot be 0, or else the number would be a 3-digit number.

Example 1.4.11 #2
How many different 4-digit numbers can be formed
using the following digits, assuming that the 4-digit number is a multiple of five?
{0, 2, 3, 5, 8}

A. 100 B. 4552 C. 4551 D. 200 E. None of these

Example
Gomer is going to order a frozen tofu cone from *I Definitely Believe It's Tofu.*
The following toppings are available:

1. carob chips
2. frosted alfala sprouts
3. seaweed sprinkles
4. rolled oats
5. rose hips

He may choose all, some or none of these toppings. How many topping combinations are possible?
What are we trying to count? Five toppings: CC, FAS, SS, RO, RH

Here are several different combinations of items.

1. RO, SS
2. FAS
3. RH, CC, FAS
4. SS, CC
5. CC, FAS, RO, RH
6. (blank: this means we didn't choose any of the toppings)
7. CC, FAS, SS, RO, RH
8. SS

How many of these are possible?
What we have is actually a ____.
If we write each of the different combination options listed above as a set, we see that
Each combination of toppings is a ___________________________ of the five-topping set, so we are just counting the ________________________________ in a set with five elements which is __________________.

Alternative solution, using FCP
We have a set of five toppings: {CC, FAS, SS, RO, RH}
We have shown that the number of different combinations of toppings is 32, because each combination of toppings is a subset of this set, and a set with 5 elements has 32 subsets.
We can also get this answer by using the FCP. When we select a combination of toppings we are actually making five ________________________________.

1. carob chips: 2 options ("yes" or "no")
2. frosted alfalfa sprouts: 2 options ("yes" or "no")
3. seaweed sprinkles: 2 options ("yes" or "no")
4. rolled oats: 2 options ("yes" or "no")
5. rose hips: 2 options ("yes" or "no")

According to the FCP, the number of outcomes is ____________________________________

"All, some, or none"

If a counting problem involves an "all, some, or none" situation, then the number of outcomes will always be a ______________________________ (such as 2, 4, 8, 16, 32, 64, 128, 256 and so on).
This is because the situation involves a series of _____________________________ decisions, so the Fundamental Counting Principle will have us _____________________________a series of twos.
This also due to the fact that such a problem is asking for the number of subsets in a particular set.

Exercise
Homerina is having a birthday party for her pet wolverine, John. John has a list of 9 gifts that he would like to receive: a duck, a hamster, a puppy, a mouse, a goldfish, a frog, a toad, a chicken, and a claw sharpener. How many combinations of gifts are possible, assuming that Homerina may buy all, or some, or none of those items?

A. 81 B. 1280 C. 512 D. 18 E. None of these

Exercise
Hjalmar, Gomer, Plato, Euclid, Socrates, Aristotle, Hjalmarina and Gomerina form the board of directors of the Lawyer and Poodle Admirers Club. They will choose from amongst themselves a Chairperson, Secretary, and Treasurer. No person will hold more than one position.
How many different outcomes are possible?

A. 336 B. 24 C. 512 D. 21

Exercise
Erasmus is trying to guess the combination to his combination lock.
The "combination" is a sequence of three numbers, where the numbers range from 1 to 12, with **no numbers repeated**.
How many different "combinations" are possible if he knows that the last number in the combination is either 1 or 11?

A. 264 B. 1320 C. 220 D. 288 E. 180

Dependent decisions

If a Fundamental Counting Principle problem involves _____________________________, and one decision involves a special condition, **the decision with the ____________________________ takes priority over the others**.

Example
The Egotists' Club has 6 members: A, B, C, D, E, and F. They are going to line up, from left to right, for a group photo. After lining up in alphabetical order (ABCDEF), Mr. F complains that he is always last whenever they do things alphabetically, so they agree to line up in reverse order (FEDCBA) and take another picture. Then Ms. D complains that she's always stuck next to Mr. C, and that she never gets to be first in line.
Finally, in order to avoid bruised egos, they all agree to take pictures for every possible left-to-right line-up of the six people. How many different photos must be taken?

A. 14 B. 720 C. 823,543 D. None of these

Similar situations
Suppose we had the same situation, but with 8 people instead of six.
Then the number of ways to arrange them in a row would be $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$.
Likewise, if there were ten people, the number of arrangements would be
$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3{,}628{,}800$ and so on.

Factorials

Numbers like
$6 \times 5 \times 4 \times 3 \times 2 \times 1$
$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
are called ______________________

"n factorial"
If **n** is a positive integer, then _________________________, denoted______________________,
is the number **n** multiplied by all the smaller positive integers.

$n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$

n! is the number of ways to arrange n objects.

Also, **$0! = 1$**

Exercise
Macbeth is trying to guess the password for Gomerina's email account. He knows that the password consists of 4 letters chosen from this set:

{g,o,m,e,r,i,n,a}.

How many passwords are possible, if a password does not contain repeated letters and the third letter is a vowel?

A. 32 B. 256 C. 840 D. 1344 E. None of these

All, Some, or None

If a counting problem involves an "all, some, or none" situation, then the number of outcomes will always be a ____. Yes we already covered this. Pay attention.
This is because the situation involves a series of _____________________________________.
This also due to the fact that such a problem is asking for the number of ______________ in a particular set.

Part 1 Module 5 - Factorials, Permutations, and Combinations

We want to be able to recognize when a counting problem can be solved directly by using ______________________________, instead of using the ________________________________.

If a counting question asks for the number of ways to arrange (put in order) ________________________, then the number of outcomes is ____________

Example: Suppose we have a collection of five books.
In how many ways can the five books be arranged in a row?
Answer: The number of ways to arrange five objects in a row is ________________________

Example #1
Select the situation where the number of outcomes is 5!
A. We can choose any combination ("all, some, or none") of pizza toppings from a list of five toppings.
B. We will take a list of five job applicants and rank them from "best" to "worst."
C. We will form a five-digit number using digits chosen from this set: {2, 4, 5, 7, 9}
D. All of these
E. None of these

Example #2
The contestants in this year's Ironic Man Octathlon must compete in all of the following events: wolverine shearing; tobacco spitting; manatee racing; squid wrestling; unicycle demolition derby; spaghetti weaving; elephant tipping; cat herding.
The organizers must decide on the order in which the events will be scheduled. How many outcomes are possible?
A. 40,320   B. 256   C. 30,240   D. None of these

Two special formulas

We are about to encounter two special formulas that can be used to quickly express the number of outcomes for certain kinds of counting problems.

These two formulas are called, respectively, the ______________________________

and the ______________________________

The Permutation Formula P(n,r)

Suppose that a counting problem involves selecting and arranging r distinct elements from a set of n elements.
We say that this is a **permutation problem**, and we say that the number of outcomes is **"the number of permutations of n things taken r at a time"**, abbreviated P(n, r).

Example: a Permutation problem
Due to time constraints, the organizers of this year's Ironic Man Octathlon are going scale things back, and produce a pentathlon, instead. From the original list of eight events (listed previously) they will select five events and then decide on the order in which the events will be scheduled.
How many outcomes are possible?

The Combination Formula C(n,r)

Suppose that a counting problem involves just **selecting,** but **not arranging** *r* distinct elements from a set of *n* elements. We say that this is a **combination problem**, and we say that the number of outcomes is **"the number of combinations of *n* things taken *r* at a time",** abbreviated C(n, r).

"Order doesn't matter…"

We say that for a **combination** problem, **"order doesn't matter."**
This means that an outcome is determined entirely by **which** elements are selected, **not** by the order in which the elements are selected or listed.
On the other hand, for a **permutation** problem, **order is important.**
That means that for a permutation problem, we care not only about which elements are selected, but also about the order in which the elements are selected or arranged.

Example: a combination problem
We have following six coins in a jar: a penny, a nickel, a dime, a quarter, a half-dollar, and a silver dollar.
Two of the coins will be randomly selected and their values will be added.
For example, if we select the dime and the penny, the monetary sum is 11¢. If we select the quarter and the silver dollar, the monetary sum is $1.25.
How many different monetary sums are possible?

Example #3
Select the counting problem in which the number of outcomes is P(7, 3).

A. Using letters from the set {a, b, c, d, e, f, g}, form a three-letter password with no repeated letters.
B. Using letters from the set {a, b, c, d, e, f, g}, form a three-letter sequence which may have repeated characters.
C. Using letters from the set {a, b, c, d, e, f, g}, form a three-letter subset.
D. All of these.
E. None of these.

Example #4

Select the counting problem in which the number of outcomes is C(5,3).

A. From a group of five waitresses, select three waitresses and tell them that they are fired.

B. From a group of five waitresses, select one waitress to bus tables, a second waitress to fold napkins, and a third waitress to polish silverware.

C. From a group of five waitresses, assign a waitress to Table #1, a waitress to table #2 and a waitress to table#3; the same waitress may get more than one assignment.

D. All of these.

E. None of these.

Guidelines

When a problem involves selecting individuals from a group:

1. If it is possible to select the same individual more than once, then it isn't a P(n,r) problem and it isn't a C(n,r) problem. It is a FCP problem.

Otherwise:

2. If the individuals are being treated differently from one other (for instance, if they are being given different jobs, titles or gifts), then order matters and it is a P(n,r) problem (unless one selection involves a special condition, in which case it is FCP).

3. If the individuals are being treated the same as one another (for instance, if they are all getting the same job, title or gift), then order doesn't matter and it is a C(n,r) problem.

Example #5

A group of ten hobbits gets together to compare rings. Prior to the meeting, each hobbit shakes hands with each of the other hobbits. How many handshakes occur?

A. 10! B. 100 C. 90 D. 45 E. None of these

Example #6

Harpo, Groucho, Chico, Zeppo, Gummo, Karl, and Skid have won three tickets to the opera. They will randomly choose three people from their group to attend the opera. How many outcomes are possible?

A. 21 B. 35 C. 210 D. 343 E. None of these

Example #7
Here is a similar problem in which ____

____.

Harpo, Groucho, Chico, Zeppo, Gummo, Karl, and Skid have won three tickets to the opera. They will randomly choose three people from their group to attend the opera.
One person gets to ride to the opera in a taxi, another gets to ride to the opera in the bed of a turnip truck, and the third person gets to ride in a rickshaw.
How many outcomes are possible?

Example #8
Select the counting problem where the number of outcomes is C(6,3). Each problem involves making three selections from this group of six people: Homer, Gomer, Homerina, Gomerina, Plato, Mrs. Plato

A. Select a president, vice-president, and treasurer. Nobody may hold more than one office.
B. Select a person to receive a pencil, select a person to receive a calculator, select a person to receive a sheet of scratch paper. It is possible that the same person might be selected to receive more than one of these gifts.
C. Select three people to take the rhinoceros for a walk.
D. Select a president, a vice-president, and a treasurer. Nobody can hold more than one office, but Gomer, who is bad at math, is not eligible to be treasurer.
E. None of these

Example #9
There are 8 hamsters in a recreational hamster league. They are going to schedule a round-robin tournament (in which each hamster will race each of the other hamsters one time). How many races will be held?

A. 256 B. 64 C. 56 D. 28 E. None of these

Example #10
Macbeth is trying to guess the password for Gomerina's email account. He knows that the password consists of 4 letters chosen from this set:
{g,o,m,e,r,i,n,a}.
How many passwords are possible, if a password does not contain repeated letters and the third letter is a vowel?

A. 32 B. 256 C. 840 D. 1344 E. None of these

Part 1 Module 6 - More counting problems

EXERCISE #1 From The FUNDAMENTALIZER, Part 2

Mrs. Plato is going to order supper at a restaurant. She will choose items from each of the following menu categories:

Category A: lettuce salad; cole slaw; clam chowder; potato salad; egg salad; vegetable soup; bean soup; macaroni salad.

Category B: lamb; grouper fillet; pork chops; prime rib; shrimp; lobster; roast beef; roast pork; fried chicken; baked chicken; broiled salmon.

Category C: baked potato; French fries; hash browns; rice; sweet potato; steamed carrots; green beans; pilaf.

How many meal combinations are possible, assuming that Mrs. Plato will choose 3 items from Category A, 3 items from Category B, and 2 items from Category C, and no item will be selected more than once?

A. 595,056 B. 18,627,840 C. 258,720 D. 43,614,208

Exercise #2 From The FUNDAMENTALIZER, Part 2

Mr. Moneybags, while out for a stroll, encounters a group of ten children. He has in his pocket four shiny new dimes and three shiny new nickels that he will give to selected children. In how many ways may these coins be distributed among the children, assuming that no child will get more than one coin?

A. 25,200 B. 3,628,800 C. 4,200 D. 604,800

Exercise #3

There are nine waitresses and six busboys employed at the trendy new restaurant The House of Hummus. From among each of these groups The International Brother/Sisterhood of Table Service Workers will select a shop steward and a secretary. How many outcomes are possible?

A. 51 B. 540 C. 2160 D. 102 E. None of these

Would we ever need to add instead of multiply?

In the examples we have looked at so far, we found various numbers, perhaps by using the permutation formula or the combination formula, and then ____

In some situations, however, we may want to ____, at the end of a calculation.

The following example suggests why we might add, rather the multiply.
EXAMPLE
Suppose we ask, "How many of you are 18 years old?" and, by show of hands, we see that there are 25 people who are 18 years of age.
Next, suppose we ask, "How many of you are 19 years old" and, by show of hands, we see that there are 30 people who are 19 years of age.

Now we want to use those results to answer the question "How many are ______________________________?"
It would make no sense to say that the number of people who are 18 or 19 years old is 25x30 = 750.

If 25 people are 18 years old, and 30 people are 19 years old, then the number of people who are 18 or 19 years old is 25+30 = 55.

This suggests the following fact:
To find the number of outcomes in an "_________________________" situation, find the number of of options for each case, and ______________________________.

More formally, if A, B are mutually exclusive conditions, then

____________________________________.

"OR" means "ADD"

Generally, in counting problems or probability problems,

"AND" means _______________

"OR" means _________________

Exercise #3a
There are nine waitresses and six busboys employed at the trendy new restaurant The House of Hummus. The International Brother/Sisterhood of Table Service Workers will either select a shop steward and a secretary from among the waitresses, or they will select a shop steward and a secretary from among the busboys. How many outcomes are possible?

A. 51 B. 540 C. 2160 D. 102 E. None of these

EXERCISE #4
A couple is expecting the birth of a baby. If the child is a girl, they will choose her first name and middle name from this list of their favorite girl's names: Betty, Beverly, Bernice, Bonita, Barbie.
If the child is a boy, they will choose his first name and middle name from this list of their favorite boy's names: Biff, Buzz, Barney, Bart, Buddy, Bert.
In either case, the child's first name will be different from the middle name.
How many two-part names are possible?

A. 50 B. 600 C. 61 D. 900

Exercise #5
The mathematics department is going to hire a new instructor. They want to hire somebody who possesses at least four of the following traits:

1. Honest 2. Trustworthy 3. Loyal 4. Gets along well with others 5. Good at math 6. Good handwriting

In how many ways is it possible to combine at least four of these traits?

A. 360 B. 15 C. 22 D. 48 E. None of these

Exercise #6
Tonight there are 8 bus boys and 7 dishwashers working at the trendy new restaurant Cap'n Krusto's Crustacean Castle. Because it is a slow night the manager will select either 3 bus boys or 2 dishwashers and send them home early. How many outcomes are possible?

A. 14112 B. 378 C. 77 D. 1176 E. None of these

Exercise #8
Among a certain group of 28 mules, 23 are stubborn, 19 are obstinate, and 16 are both stubborn and obstinate.
How many are stubborn or obstinate?

A. 26 B. 51 C. 10 D. None of thes

Exercise #9
In the Psychology Department there are 8 faculty members. 2 faculty members will be chosen to design a new course offering and 3 other faculty members will be chosen to hang out in the faculty parking lot to prevent students from parking there. How many different outcomes are possible?

A. 560 B. 48 C. 240 D. None of these.

New Problem Type

In the Psychology Department there are 8 faculty members.
They will **either** choose 2 faculty members to design a new course offering **or** choose 3 faculty members to hang out in the faculty parking lot to prevent students from parking there. How many different outcomes are possible?