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> restart; read "/Users/heba/Desktop/Implementations/Absolute_Factorization.txt" :
> with(LREtools) :

$$\begin{aligned} \text{_Env\_LRE\tauau} &:= \tauau; \\ \text{_Env\_LRE\_x} &:= x; \end{aligned}$$


$$\begin{aligned} \text{_Env\_LRE\tauau} &:= \tau \\ \text{_Env\_LRE\_x} &:= x \end{aligned} \quad (1)$$

> AbsFactorization( $\tau^4 + \tauau + x$ );

$$true \quad (2)$$

>  $L := (16 \cdot x + 32) \cdot \tau^4 + (-16 \cdot x - 40) \cdot \tau^3 + (4 \cdot x + 12) \cdot \tau^2 - 2 \cdot (2 \cdot x + 3) \cdot (x + 3) \cdot (x + 2) \cdot \tau^1 - (x + 3) \cdot (x + 1) \cdot (x + 2)^2 \cdot x;$ 

$$L := (16x + 32)\tau^4 + (-16x - 40)\tau^3 + (4x + 12)\tau^2 - 2(2x + 3)(x + 3)(x + 2)\tau - (x + 3)(x + 1)(x + 2)^2x \quad (3)$$

> AbsFactorization( $L$ );

$$[2, \{\tau^2 + (-x - 2)\tau - (x + 2)(x + 1)^2x, 16\tau^2 + (-16x - 24)\tau - (2x + 3)^2(2x + 1)^2\}] \quad (4)$$

Trying the algorithm from section 3.3. This uses the Hom program:
> read "/Users/heba/Desktop/Implementations/Hom.txt" :
> AbsFactorization( $L$ , 'Hom method');

$$[2, \{-\tau^2 + (x + 2)\tau + (x + 2)(x + 1)^2x, 16\tau^2 + (-16x - 24)\tau - (2x + 3)^2(2x + 1)^2\}] \quad (5)$$

OEIS A260772 example
>  $L := (x + 5) \cdot (x + 4) \cdot (25 \cdot x^2 + 130 \cdot x + 141) \cdot \tau^4 - 30 \cdot (x + 4) \cdot (7 \cdot x + 13) \cdot \tau^3 + (-1100 \cdot x^4 - 12320 \cdot x^3 - 48664 \cdot x^2 - 80740 \cdot x - 47400) \cdot \tau^2 + 120 \cdot (x + 6) \cdot (x + 1) \cdot \tau - 16 \cdot (x + 1) \cdot (25 \cdot x^2 + 180 \cdot x + 296) \cdot x;$ 

$$L := (x + 5)(x + 4)(25x^2 + 130x + 141)\tau^4 - 30(x + 4)(7x + 13)\tau^3 + (-1100x^4 - 12320x^3 - 48664x^2 - 80740x - 47400)\tau^2 + 120(x + 6)(x + 1)\tau - 16(x + 1)(25x^2 + 180x + 296)x \quad (6)$$

> AbsFactorization( $L$ );

$$[2, \{(2x + 5)(5x + 3)(x + 2)\tau^2 + (-440x^3 - 1584x^2 - 1780x - 600)\tau - 16(5x + 8)(2x + 1)x, (2x + 5)(10x + 9)(x + 2)\tau^2 + (-880x^3 - 3432x^2 - 4220x - 1650)\tau - 16(10x + 19)(2x + 1)x\}] \quad (7)$$

> AbsFactorization( $L$ , 'Hom method');

$$[2, \{(2x + 5)(5x + 3)(x + 2)\tau^2 + (-440x^3 - 1584x^2 - 1780x - 600)\tau - 16(5x + 8)(2x + 1)x, -(2x + 5)(10x + 9)(x + 2)\tau^2 + (880x^3 + 3432x^2 + 4220x + 1650)\tau + 16(10x + 19)(2x + 1)x\}] \quad (8)$$


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