Quiz 1 **MAP 2302/3305 – Ordinary Differential Equations**

Student's Name: Solutions

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This is a 50 minute quiz.

1. Solve the differential equation

$$ty' + y = 2t + 1$$

given the initial condition $y(1) = 5$. Find the largest interval on which the solution $y(t)$ exists.

standard form: $y' + \frac{1}{t}y = 2 + \frac{1}{t}$

$\boxed{\text{interval}: 0 < t < \infty}$

Integrating factor: $\mu(t) = e^{\int \frac{1}{t}dt} = e^{\ln|t|}$

$= t$ (since $t > 0$)

Leibnitz method:

$\frac{d}{dt}(ty^*) = (2 + \frac{1}{t}) \cdot t = 2t + 1 \Rightarrow ty^* = t^2 + t + C$

$\Rightarrow y = t + 1 + \frac{C}{t}$

$5 = y(1) = 2 + C$

$\Rightarrow \boxed{y = t + 1 + \frac{3}{t}}$

2. Find the general solution to the differential equaton

$$\frac{dy}{dx} = x\sqrt{y+1}.$$

Find the solution curve that passes through the point $(x_0, y_0) = (2, 3)$.

$\frac{1}{\sqrt{y+1}} \frac{dy}{dx} = x$

$\int \frac{dy}{\sqrt{y+1}} - \int x\, dx = C$

$\boxed{2\sqrt{y+1} - \frac{x^2}{2} = C}$ general sol.

$2 = 4 - 2 = 2\sqrt{3+1} - \frac{4}{2} = C$

$\boxed{2\sqrt{y+1} - \frac{x^2}{2} = 2}$ or $\boxed{y = \left(1 + \frac{x^2}{4}\right)^2 - 1}$

specific solution

3. A certain college graduate borrows \$8000 to buy a car. The lender charges interest at an annual rate of 10%. Assume that the interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k.

(a) Set up the differential equation.

$$\boxed{\begin{aligned}\frac{dS}{dt} &= rS - k \\ &= \frac{1}{10}S - k\end{aligned}}$$

(b) Find the general form of the solution.

$$S = Ce^{rt} + 10k$$

$$S(0) = 8000$$
$$= C + 10k$$

$$\boxed{S(t) = (8000 - 10k)e^{\frac{t}{10}} + 10k}$$

(c) How large does k have to be so that the loan is eventually paid off?

We need $8000 - 10k \leq 0$

So $\boxed{k > 800}$