Quiz 1

MAP 2302/3305 – Ordinary Differential Equations

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This is a 50 minute quiz.

1. Solve the differential equation

\[ ty' + y = 2t + 1 \]

given the initial condition \( y(1) = 5 \). Find the largest interval on which the solution \( y(t) \) exists.

- **Standard form:** \( y' + \frac{1}{t} y = 2 + \frac{1}{t} \)
- **Integrating factor:** \( \mu(t) = e^{\int \frac{1}{t} dt} = e^{\ln t} = t \)
- **Leibnitz method:**

  \[ \frac{d}{dt}(ty^*) = (2 + \frac{1}{t}) \cdot t = 2t + 1 \]

  \( \Rightarrow \)

  \[ ty^* = t^2 + t + C \]

  \( \Rightarrow \)

  \[ y = \frac{t^2 + t + C}{t} \]

  \( \Rightarrow \)

  \[ y = t + 1 + \frac{C}{t} \]

  \( \Rightarrow \)

  \[ y = t + 1 + \frac{3}{t} \]

2. Find the general solution to the differential equation

\[ \frac{dy}{dx} = x \sqrt{y + 1} \]

Find the solution curve that passes through the point \((x_0, y_0) = (2, 3)\).

\[ \frac{1}{\sqrt{y + 1}} \cdot \frac{dy}{dx} = x \]

\[ \int \frac{dy}{\sqrt{y + 1}} = \int x \, dx = C \]

\[ 2\sqrt{y + 1} - \frac{1}{2}x^2 = C \]  \( \text{general sol.} \)

\[ 2 = 4 - 2 = 2\sqrt{3} + 1 - \frac{1}{2} = \frac{1}{2} \]

\[ 2\sqrt{y + 1} - \frac{1}{2}x^2 = 2 \]

or

\[ y = \left(1 + \frac{x^2}{4}\right)^2 - 1 \]

< Specific solution >
3. A certain college graduate borrows $8000 to buy a car. The lender charges interest at an annual rate of 10%. Assume that the interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate $k$.

(a) Set up the differential equation.

\[
\frac{ds}{dt} = rs - k = \frac{1}{10} s - k
\]

(b) Find the general form of the solution.

\[
s = Ce^{rt} + 10k
\]

\[
s(0) = 8000
\]

\[
= C + 10k
\]

\[
s(t) = (8000 - 10k)e^{\frac{t}{10}} + 10k
\]

(c) How large does $k$ have to be so that the loan is eventually paid off?

We need $8000 - 10k \leq 0$

So $k \geq 800$