Quiz 2 **MAP 2302/3305 – Ordinary Differential Equations**

Student's Name: Solutions

E. Hironaka

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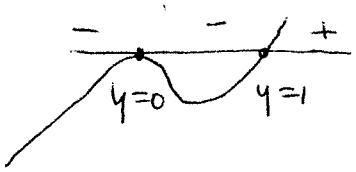
This is a 50 minute quiz.

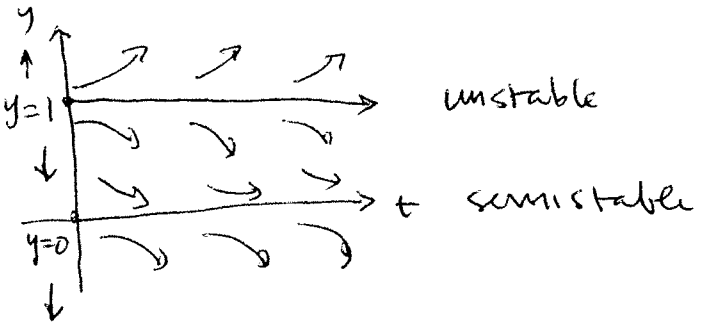
1. Draw a graph that describes the equilibrium solutions and flows for the differential equation:

$$\frac{dy}{dt} = y^2(y-1) \qquad -\infty < y_0 < \infty.$$

Label the stable, semi-stable and unstable equilibrium solutions.

f'(y) - graph





2. Solve the differential equation

$$y'' - 3y' + 2y = 0,$$

given the initial conditions $y(0) = 5$ and $y'(0) = 3$. What happens to the solution $y(t)$ as $t \to \infty$? Draw a rough graph of $y(t)$.

$r^2 - 3r + 2 = 0$

$(r-2)(r-1) = 0$

General solution

$y = c_1 e^{2t} + c_2 e^{t}$

$5 = c_1 + c_2$

$-3 = 2c_1 + c_2$

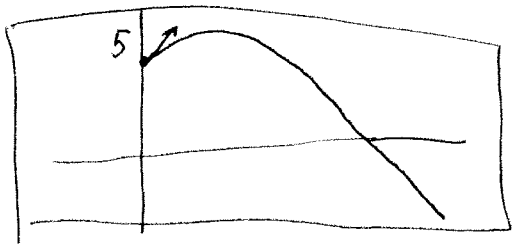
$2 = -c$

$c_1 = -2 \qquad c_2 = 7$

Specific solution

$y = -2e^{2t} + 7e^{t}$

$\lim_{t \to \infty} y(t) = -\infty$



3. Consider the differential equation $y'' + 9y = 0$.

(a) Show that $y_1(t) = \cos(3t)$ and $y_2(t) = \sin(3t)$ satisfy the differential equation.

$y_1' = -3\sin(3t)$ $\quad$ $y_2' = 3\cos(3t)$

$y_1'' = -9\cos(3t)$ $\quad$ $y_2'' = -9\sin(3t)$

$y_1'' + 9y_1 = -9\cos(3t) + 9\cos(3t) = 0$ ✓

$y_2'' + 9y_2 = -9\sin(3t) + 9\sin(3t) = 0$ ✓

(b) Use the Wronskian to determine whether y_1 and y_2 (from part 3(a)) are linearly independent. Do they form a set of fundamental solutions?

$$W(t) = \begin{vmatrix} \cos(3t) & \sin(3t) \\ -3\sin(3t) & 3\cos(3t) \end{vmatrix}$$

$$= 3\cos^2(3t) + 3\sin^2(3t) = 3 \neq 0$$

$\Rightarrow$ yes, they are linearly indep

$\Rightarrow$ yes, they are a set of fundamental solutions for $y'' + 9y = 0$

4. (Bonus) Find a differential equation that has fundamental solutions $y_1(t) = e^{7t}$ and $y_2(t) = e^{-3t}$.

$(r-7)(r+3)$

$= r^2 - 4r - 21$

$y'' - 4y' - 21y = 0$

has fundamental solutions $y_1 = e^{7t}$ $\quad$ $y_2 = e^{-3t}$