Quiz 3 **MAP 2302/3305 – Ordinary Differential Equations**

Student's Name: Solutions

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1. Use variation of parameters to find the general solution to

$$y'' - y' - 2y = 2e^{-t}$$

given that $y_1(t) = e^{2t}$ and $y_2(t) = e^{-t}$ is a fundamental set of solutions.

$$W(y_1,y_2) = \begin{vmatrix} e^{2t} & e^{-t} \\ 2e^{2t} & -e^{-t} \end{vmatrix} = -e^{t} - 2e^{t} = -3e^{t}$$

$$u_1' = \frac{-y_2 g}{W} = \frac{-e^{-t}\cdot 2e^{-t}}{-3e^{t}} = \frac{2}{3}e^{-3t} \qquad u_1 = -\frac{2}{9}e^{-3t}$$

$$u_2' = \frac{y_1 g}{W} = \frac{e^{2t}\cdot 2e^{-t}}{-3e^{t}} = -\frac{2}{3} \qquad u_2 = -\frac{2}{3}t$$

$$\boxed{Y = -\frac{2}{9}e^{-3t}\cdot e^{2t} - \frac{2}{3}te^{-t} = -\frac{2}{9}e^{-t} - \frac{2}{3}te^{-t}}$$

$$\boxed{y = c_1e^{2t} + c_2e^{-t} - \frac{2}{3}te^{-t}}$$

2. A mass weighing 1 lb stretches a spring 2 ft. Assume there is no damping and that the mass is pulled down an additional $a > 0$ inches and released. Assume that the gravitational constant is 32 ft/sec^2.

(a) Give a formula for the position of the mass as a function depending on a.

$g=32$

$L=2$

$$1 = W = mg = m\cdot 32 \Rightarrow m = \frac{1}{32}$$

$$1 = kL = 2k \Rightarrow k = \frac{1}{2}$$

Diff'eqn. $\frac{1}{32}u'' + \frac{1}{2}u = 0$

or $u'' + 16u = 0$

$u(0) = a = c_1$

$u'(0) = 0 = c_2$

general sol: $u = c_1\cos 4t + c_2\sin 4t$

$$\boxed{u = a\cos 4t}$$

(b) Find the frequency and period. Does it depend on a?

$$\boxed{\text{frequency} = 4}$$

$$\boxed{\text{period} = \frac{2\pi}{4} = \frac{\pi}{2}.}$$

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3. Find the recurrence relation and the first four terms in each of the linearly independent solutions to

$$y'' + y = 0,$$

where y is the Taylor series expansion around $x_0 = 0$.

$$y = \sum_{n=0}^{\infty} a_n x^n \qquad y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \qquad y'' = \sum_{n=2}^{\infty} n(n-1) x^{n-2}$$

$$y'' + y = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0}^{\infty} a_n x^n$$

recurrance rel. $\boxed{a_{n+2} = -\frac{a_n}{(n+1)(n+2)}}$

$$a_0 \left(1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6}\right. + a_1 \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right.$$

Bonus. Find the Taylor series for

$$\frac{1}{2-x}$$

around $x_0 = 3$.

$$\frac{1}{2-x} = \frac{1}{-1-(x-3)} = -\frac{1}{1+(x-3)}$$

$$= -\sum_{n=0}^{\infty} (-1)^n (x-3)^n$$

$$\boxed{= \sum_{n=0}^{\infty} (-1)^{n+1} (x-3)^n}$$