

Quiz 4 **MAP 2302/3305 – Ordinary Differential Equations**

Student's Name: _____

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This is a 50 minute quiz.

1. Find the recurrence relation for series solutions to

$$y'' + xy' + 2y = 0$$

around the point $x_0 = 0$, and find $y''(0)$ and $y'''(0)$ given that $y(0) = 1$ and $y'(0) = 3$.

2. Consider the differential equation

$$(x + 1)^2 x^2 y'' - 2(x + 1)y' + x^2 y = 0 \quad (*)$$

- (a) Find the singular points of (*) and determine which ones are regular.
- (b) Find a lower bound for the radius of convergence of series solutions to (*) near the point $x_0 = 3$.

3. Consider Euler's equation

$$x^2 y'' + 2xy' + y = 0.$$

(a) Find the general solution valid on the interval $x > 0$.

(b) Find the limit

$$\lim_{x \rightarrow \infty} y(x)$$

where $y(x)$ is any solution, and briefly explain why it doesn't depend on initial conditions.

Bonus. Find a homogeneous linear second order differential equation and initial conditions on $y(1)$ and $y'(1)$ that has the particular solution

$$y = x \ln x, \quad x > 0$$

and no other.