Quiz 4 MAP 2302/3305 – Ordinary Differential Equations

Student's Name: Solutions

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This is a 50 minute quiz.

1. Find the recurrence relation for series solutions to

$$y'' + xy' + 2y = 0$$

around the point $x_0 = 0$, and find $y''(0)$ and $y'''(0)$ given that $y(0) = 1$ and $y'(0) = 3$.

a_0

$$y'' \quad \sum_{n=2}^{\infty} (n-1)n\, a_n x^{n-2} + x \sum_{n=1}^{\infty} n\, a_n x^{n-1} + 2 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$= \sum_{n=0}^{\infty} \Big((n+1)(n+2)\, a_{n+2} + n\, a_n + 2 a_n \Big) x^n$$

$$\Rightarrow \boxed{a_{n+2} = \frac{-a_n}{n+1}} \quad n = 0, 1, 2, \ldots$$

$y''(0) = a_2 \cdot 2! = -\frac{a_0}{1} \cdot 2!$

$= \boxed{-2}$

$y'''(0) = a_3 \cdot 3! = -\frac{a_1}{2} \cdot 3!$

$= -3 \cdot 3 = \boxed{-9}$

2. Consider the differential equation

$$(x+1)^2 x^2 y'' - 2(x+1)y' + x^2 y \qquad (*)$$

(a) Find the singular points of $(*)$ and determine which ones are regular.

singular points: 0, not regular since order is 2 at 0 ($\frac{-2(x+1)}{(x+1)^2 x^2}$)

-1, regular since $\frac{-2(x+1)}{(x+1)^2 x^2}$ has ord 1 at -1; $\frac{x^2}{(x+1)^2 x^2}$ has ord at -1

(b) Find a lower bound for the radius of convergence of series solutions to $(*)$ near the point $x_0 = 3$.

lower bound = min distance of x_0 to a singular point

$= \boxed{3}$