

Test 1 **MAP 2302/3305 – Ordinary Differential Equations**

Student's Name:_____

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May 30, 2008

This is an hour and 15 minute in-class test. You may use scrap paper, but please show all work on this sheet.

1. Suppose a population P is modeled by the differential equation:

$$P' = (P - 200)(100,000 - P).$$

- (a) Plot the equilibrium solutions and determine which are stable, unstable or semi-stable.

- (b) For which initial values will the population stay bounded and not eventually go extinct?

2. Solve

$$y'' + 4y' + 5y = 0,$$

with $y(0) = 1$ and $y'(0) = -3$ and sketch a graph of your solution.

3. Consider the differential equation

$$t^2 y'' - 4ty' + 6y = 0.$$

This has solution $y_1 = t^3$. Find another solution y_2 and verify that it is linearly independent from y_1 by computing the Wronskian.

4. Suppose a salt water tank has 100 gallons of capacity, and a salt water solution containing 0.1 lbs of salt per gallon. flows into the tank at the rate of 10 gallons per minute. Suppose the salt water in the tank is well mixed and exits at the same rate as it enters.

(a) Find a differential equation for the amount of salt in the tank.

(b) If the tank starts with pure water, give a formula for the amount of salt in the tank at time t .

(c) Draw a graph of the amount of salt in the tank, indicating asymptotes.

5. Find the general solution to the non-homogeneous equation

$$y'' + 3y' + 2y = 10e^{3t}.$$

(Hint: two homogeneous solutions are $y_1 = e^{-t}$ and $y_2 = e^{-2t}$.)

Bonus. Find a second order, linear, homogenous differential equation satisfied by the function

$$y(t) = e^t(5 - 2t).$$

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