Test 2 MAP 2302/3305 – Ordinary Differential Equations

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This is an hour and 15 minute in-class test. Please show all work on this sheet.

1. Find the recurrence relation for the series solution to the differential equation:

$$y'' + 4y = 0.$$

Write down the first four terms of the series solution, given that $y(0) = 1$ and $y'(0) = 1$.

2. Use Euler's method to find the general solution to

$$x^2y'' + 3xy' + 4y = 0$$

valid in the interval $x > 0$. Sketch a graph of the solution.

3. Find the Laplace transforms of the functions

(a) $5 + e^t \cos 2t$.

(b) $u_3(t)\ e^t$

4. Consider the function

$$f(t) = \begin{cases} 0 & \text{for } 0 \leq t < 1 \\ t - 1 & \text{for } 1 \leq t < 2 \\ t^2 - 4t + 5 & \text{for } 2 \leq t \end{cases}$$

(a) Sketch a graph of $f(t)$ noting concavity.

(b) Write $f(t)$ using the basic step functions $u_1(t)$ and $u_2(t)$.

(c) Find the Laplace transform of $f(t)$.

5. Use Laplace transforms to solve the differential equation, given the initial conditions.

$$y'' + 3y' + 2y = \delta(t-2) \qquad y'(0) = y(0) = 0.$$

Sketch a graph of the solution.

Bonus: Find the order of the singularity 0 for the functions: $f(x) = \ln x$, and $f(x) = e^{1/x}$. (Hint: Use L'Hôpital's rule.)