THE DYNAMICS OF MAPPING CLASSES ON SURFACES TOKYO INSTITUTE OF TECHNOLOGY – WINTER 2011-2012

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Summary:

These notes are based on twelve lectures given at the Tokyo Institute of Technology during the winter session of 2011-2012. The lectures series was a survey of mapping classes defined on orientable compact surfaces, with particular focus on pseudo-Anosov mapping classes and their dilatations.

A homeomorphism from a surface to itself considered up to isotopic equivalence is called a mapping class. Under the Nielsen-Thurston classification, these fall into three types: *periodic, reducible, and pseudo-Anosov.* Each type is distinguished by the action of the mapping class on isotopy types of simple closed curves on the surface. Our focus is on pseudo-Anosov mapping classes. These are the mapping classes such that the images of an essential simple closed curve under iterations of the map have exponentially increasing length. The growth rate is called the dilatation of the pseudo-Anosov mapping class. For a fixed surface, the set of dilatations that can occur forms a set of algebraic integers of bounded degree. We investigate properties of these algebraic integers and properties of the mapping classes that admit small dilatation.

We develop several tools in this survey. The first tool is traintracks, and digraphs. These can be used, for example, to deduce number theoretic properties of dilatations. The second tool involves combinatorial constructions of pseudo-Anosov mapping classes using graphs and plumbing techniques. The invariants of mapping classes built this way are related to properties of the graphs and their associated Artin and Coxeter groups. The third tool is Thurston's theory of fibered faces of a hyperbolic 3-manifold. Fibered faces allow us to look at whole families of mapping classes with related dynamics defined on surfaces of varying genus. We investigate the structure of mapping classes on fibered faces and how they vary as one moves in a fibered face.