Final Review **MGF 3301 Intro. to Adv. Math**

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Spring 2014

The Final Exam will be cumulative, and will include material from the first midterm review as well as what appears below.

1. Give examples of the following, and justify your answers.
   (a) A partially ordered infinite set that is not totally ordered.
   (b) A totally ordered infinite set that is not well-ordered.
   (c) A set that is countably infinite, but not equal to the natural numbers.
   (d) A set that is uncountably infinite, but not equal to the real numbers.

2. For each of the following, find functions $f : A \to B$ and $g : B \to C$ between infinite sets A, B and C, satisfying the conditions or show that it is impossible.
   (a) g is not one-to-one, but $g \circ f$ is one-to-one.
   (b) f is not onto, but $g \circ f$ is onto.
   (c) f and g are not bijections, but $g \circ f$ is a bijection.
   (d) g is not onto, but $g \circ f$ is onto.
   (e) f is not one-to-one, but $g \circ f$ is one-to-one.

3. Let a, b, c be elements of $\mathbb{N}$. For each of the following, prove or give a counter-example. You may assume that if $d = \gcd(a, b)$, then there are integers $r, s \in \mathbb{Z}$ so that
$$ra + sb = d.$$
   (a) $\gcd(\gcd(a, b), c) = \gcd(a, \gcd(b, c))$.
   (b) $\text{lcm}(\text{lcm}(a, b), c) = \text{lcm}(a, \text{lcm}(b, c))$.
   (c) If c divides a and b, then c divides $\gcd(a, b)$.
   (d) $\text{lcm}(a, b)$ divides ab.
   (e) If $\gcd(a, b) = 1$, then $\text{lcm}(a, b) = ab$.
   (f) If $\gcd(a, b) = 1$, then $[a]$ is invertible in the integers modulo b.

4. Do the following modular arithmetic problems.

   (a) Find the invertible elements of $\mathbb{Z}/24\mathbb{Z}$, and find their inverses.

   (b) Find all the powers of $[2]$ in $\mathbb{Z}/15\mathbb{Z}$.

   (c) Find all the powers of $[3]$ in $\mathbb{Z}/15\mathbb{Z}$.

   (d) In $\mathbb{Z}/n\mathbb{Z}$ show that if $[a] = [a']$ and $[b] = [b']$, then $[a+b] = [a'+b']$ and $[ab] = [a'b']$.