Midterm Review **MGF 3301 Intro. to Adv. Math**

E. Hironaka
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The midterm exam will be closed book and closed notebook. You will work on your own. Here is a sample of the kinds of questions you will be asked.

1. Give an example of each of the following from every day and from arithmetic.
   (a) A mathematical sentence with a free variable.
   (b) A mathematical sentence with an existential quantifier.
   (c) A mathematical sentence with a universal quantifier.
   (d) A mathematical sentence stating an implication.
   (e) The negations of each of the above.

2. Show using truth tables each of the following. Explain what the statement is saying in words.
   (a) $A \Rightarrow B$ is equivalent to $B \vee (\sim A)$.
   (b) $\sim (A \Rightarrow B$ is equivalent to $\sim B \wedge A$.
   (c) $\sim (A \wedge B)$ is equivalent to $(\sim A) \vee (\sim B)$.
   (d) $A \Rightarrow (\sim A)$ is always false.
   (e) $(A \wedge B) \vee (\sim A) \vee (\sim B)$ is always true.
   (f) $(\sim A) \Rightarrow (A \Rightarrow B)$ is always true.

3. Define and give an example of each of the following.
   (a) A subset of a set.
   (b) The complement of a set in a larger set.
   (c) A relation on a set.
   (d) A partial order on a set.
   (e) A total ordering on a set.

4. Prove or give a counter example.

   (a) If $A \subseteq B$, then $B^c \subseteq A^c$.
   (b) If $A \subseteq B$ and $C \subseteq D$, then $A \cup C \subseteq B \cup D$.
   (c) $B \setminus (B \setminus A) = A \cap B$.
   (d) $A \cup (B \cap C) = (A \cup B) \cap C$.

5. Which of the following relations are reflexive, symmetric, anti-symmetric, transitive?

   (a) $A = \{a, b, c\}$, $R = \{(a, a), (a, b), (b, c), (a, c)\}$.
   (b) $A = \mathbb{Z}$, $R = \{(a, b) \ : \ a \text{ and } b \text{ are both even}\}$.
   (c) $A = \mathbb{R}$, $R = \{(x, y) \ : \ xy = 1\}$.
   (d) $A = \mathbb{R}$, $R = \{(x, y) \ : \ y = x + 1\}$.

6. Give an example of a relation on the set of students in the classroom with the given property

   (a) reflexive and neither symmetric nor anti-symmetric
   (b) symmetric but not reflexive
   (c) a partial ordering but not a total ordering
   (d) transitive but not anti-symmetric

7. Prove using induction.

   (a)
   $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

   (b)
   $$\sum_{i=1}^{n} a^i = \frac{a^n - 1}{a - 1}.$$