Equivariant subsets of Peano continua under p-adic actions

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University of Florida

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Introduction

Hilbert's Fifth Problem (1900)

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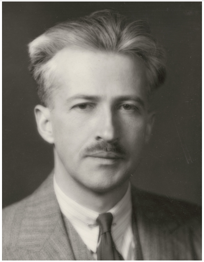

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photo courtesy of University Archives, Columbia University in the City of New York (for $20)

Definitions and Notation

Definition: *p*-adic group

For a given prime number p, the p-adic group is an abelian group of the form $A_p := \varprojlim\{\mathbb{Z}_{p^k}, \phi_k^{k+1}\}$ where $\mathbb{Z}_{p^k} := \mathbb{Z}/p^k\mathbb{Z}$, and $\phi_k^{k+1} : \mathbb{Z}_{p^{k+1}} \to \mathbb{Z}_{p^k}$ are p-fold group homomorphisms. Let $\tau \in A_p$ denote an element topologically generating A_p.

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Definition: Free Action (a.k.a. Strongly Effective Action)

We say G acts freely on the space X if and only if for every $g \in G \setminus \{e\}$ and for every point $x \in X$, we have $g(x) \neq x$.

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LEJ Brouwer
(voutsadakis.com)

Bela Kerékjártó
(history.mcs.st-and.ac.uk)

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John Pardon
(paw.princeton.edu)

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- dim M $=$ 2 (L.E.J. Brouwer 1919, separately B. Kerékjártó)
- dim M $=$ 3 (J. Pardon 2011)
- The group A_p acts by diffeomorphisms (Bochner-Montgomery 1946)
- The group A_p acts by Lipschitz homeomorphisms (Ščepin-Repovš 1997), et al.

Salomon Bochner

Deane Montgomery

(pictures from history.mcs.st-and.ac.uk)

Evengy Ščepin and

Dušan Repovš

(from Jed Keesling and www.pef.uni-lj.si)

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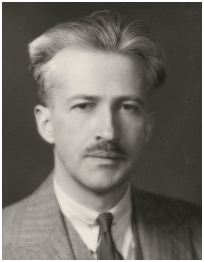

Paul Althaus Smith
(hey for $20 I'm going to use it more than once)

photo of P.A. Smith courtesy of University Archives, Columbia University in the City of New York

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- $\dim_{\mathbb{Z}} M/A_p \neq \dim(M)$ (P.A. Smith 1940)
- $\dim_{\mathbb{Z}} M/A_p = 2 + \dim(M)$ (C.T. Yang 1960)

(math.upenn.edu)

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- The quotient space ${}^{M}/_{A_p}$ cannot be dimensionally full-valued (Raymond-Williams 1963)

Frank Raymond
(math.lsa.umich.edu)

Robert Fones Williams
(ma.utexas.edu)

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A proof of the last can be found in A. N. Dranishnikov's "On free actions of zero-dimensional compact groups" (1988).

A. N. Dranishnikov
(math.ufl.edu)

My results

Theorem (Keesling, Maissen, Wilson)

For each prime p, there is only one free action by A_p on the space of irrational numbers.

James Keesling

David C. Wilson

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Theorem (Keesling, Maissen, Wilson)

The Hilbert-Smith conjecture is equivalent to asserting that the unique p-adic group action on the space of irrational numbers cannot be extended to a manifold compactification.

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Theorem (Bing 1949)

Every Peano continuum is partitionable.

R. H. Bing
(history.mcs.st-and.ac.uk)

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Lemma (Maissen)

Let X and Y be Peano continua. If $f : X \to Y$ is a perfect, light open map and $U \subset Y$ is open, with the property that $\bar{U}$ is compact and locally connected, then $f^{-1}(\bar{U})$ is also compact and locally connected.

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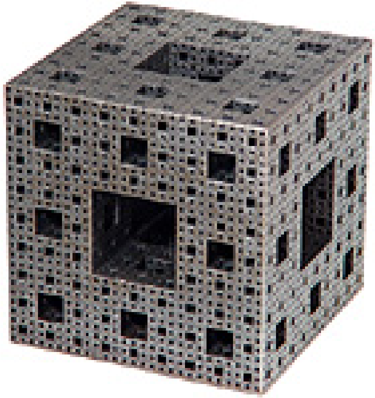
For fun, I will construct such a Menger curve out of p-adic solenoids.

Menger Curve

Karl Menger
(en.wikipedia.org)

Menger Curve



Menger Curve μ^1
(a.k.a. Menger Sponge)
(image from mathworld.wolfram.com)

Karl Menger
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solenoids

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A p-adic solenoid is a continuum of the form: $\Sigma_p := \varprojlim\{S^1, \phi_n^{n+1}\}$ where each $\phi_n^{n+1} : S^1 \to S^1$ is a p-to-1 covering map.

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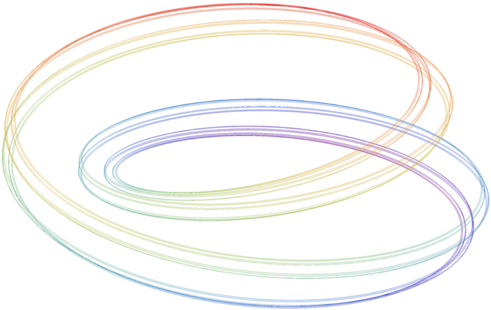

(Dyadic Solenoid)

(taken from en.wikipedia.org)

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For a given choice of the point $x \in X$, let us see what the full orbit of the arc $J \subset X$ from x to $y \in X$ looks like, depending upon our choice of the point y.

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Taking $y \notin A_p(x)$ we obtain a sub-space $Z \subset X$

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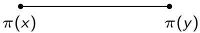

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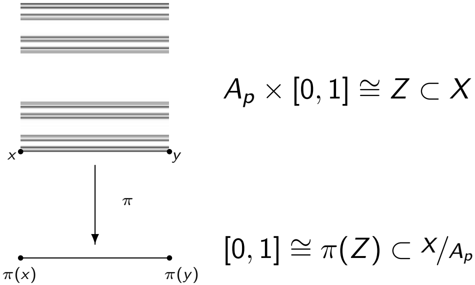

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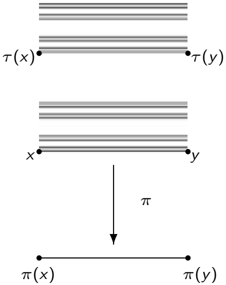

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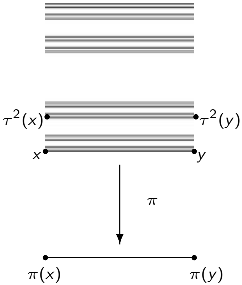

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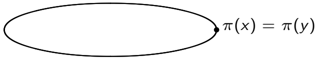

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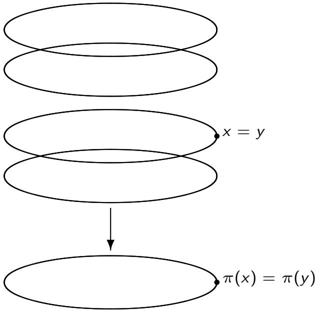

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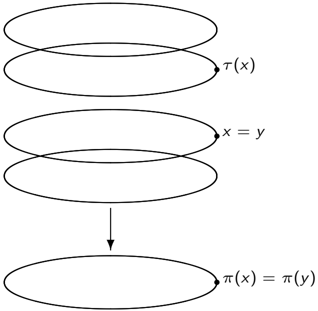

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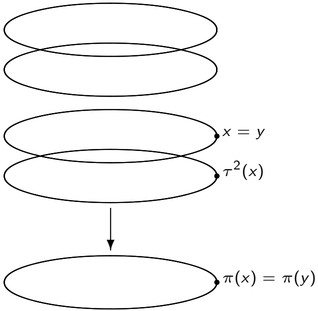

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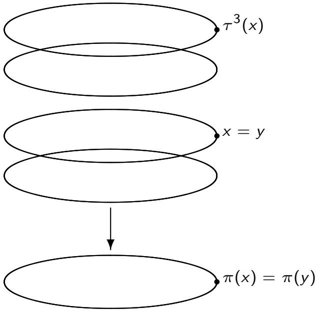

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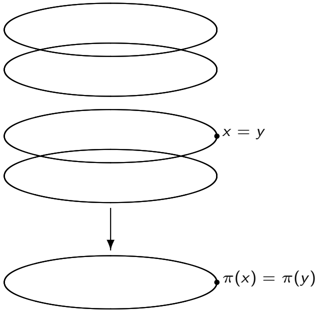

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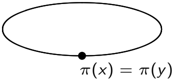

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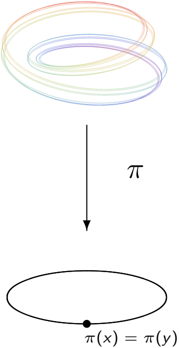
If $\tau \in A_p$ generates A_p, then by taking $y = \tau(x)$ we obtain an invariant sub-space $Z \subset X$ with $Z \cong \Sigma_p$ the p-adic solenoid.



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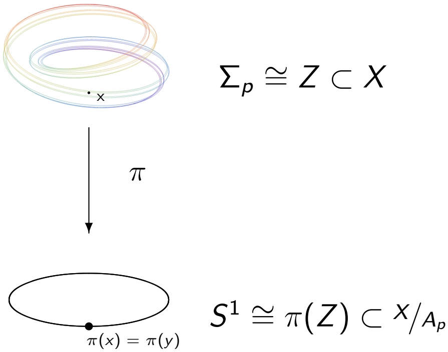
If $\tau \in A_p$ generates A_p, then by taking $y = \tau(x)$ we obtain an invariant sub-space $Z \subset X$ with $Z \cong \Sigma_p$ the p-adic solenoid.



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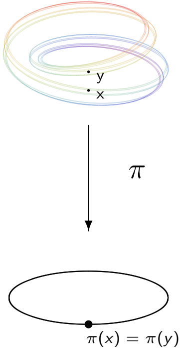

$\Sigma_p \cong Z \subset X$

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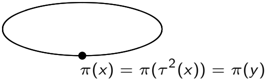

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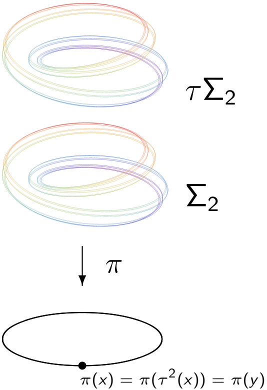

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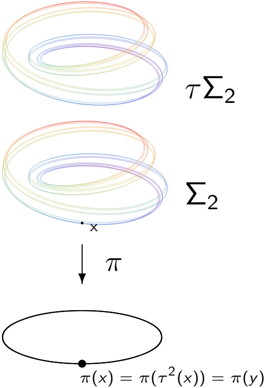

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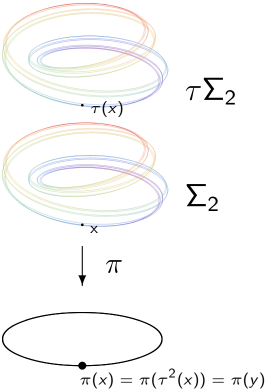

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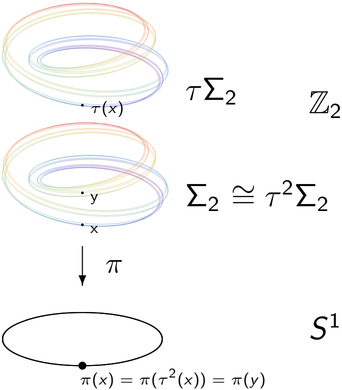

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Properties of solenoids

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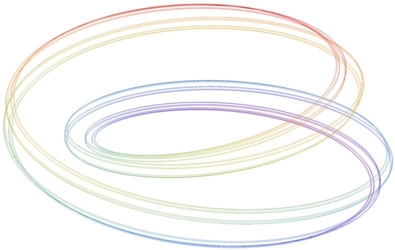

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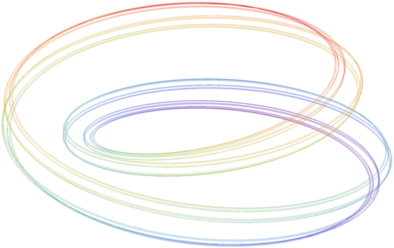

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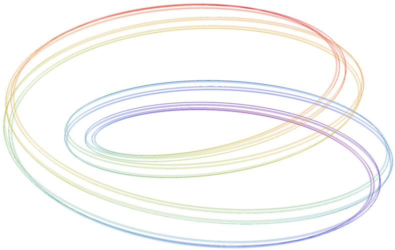

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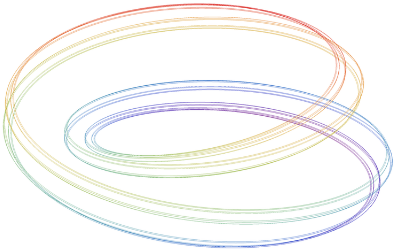

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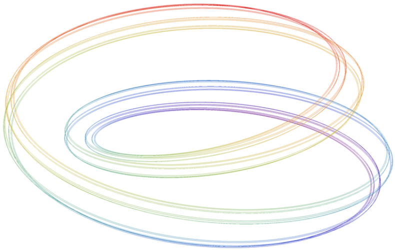

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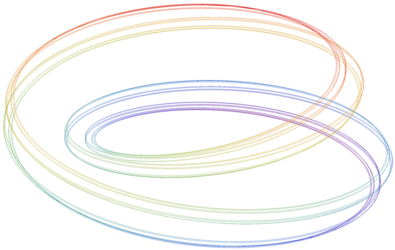

Characterization of the Menger Curve, μ^1

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Richard Davis Anderson
(maa.org)

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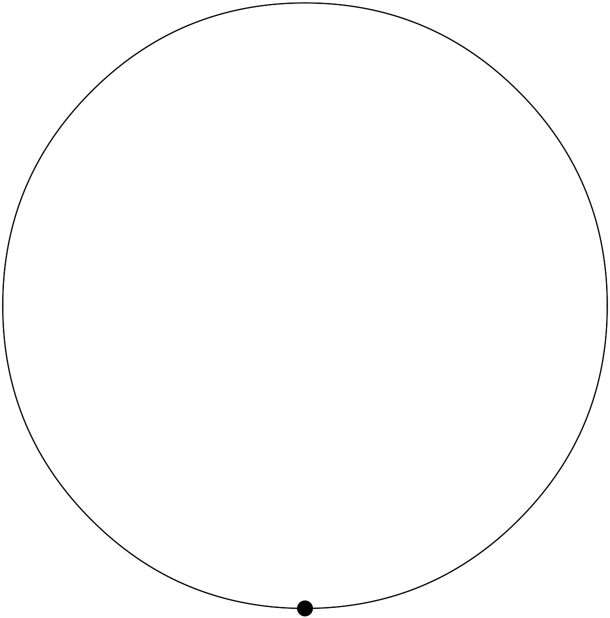
The construction: the view from the quotient space X/A_p



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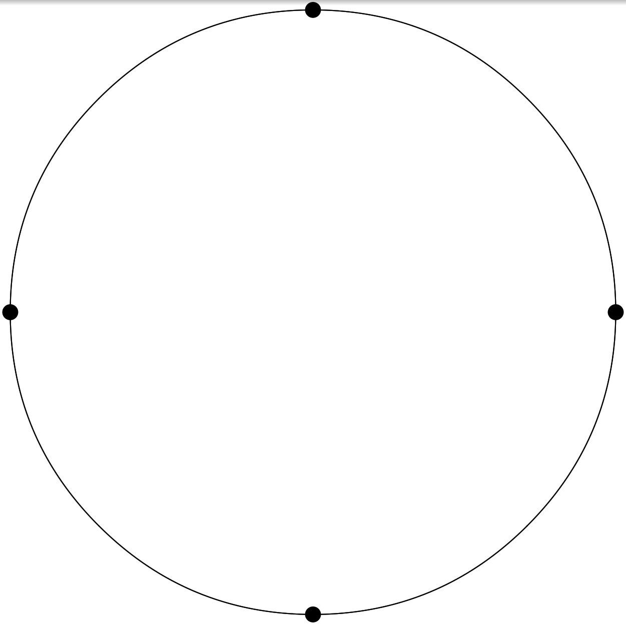

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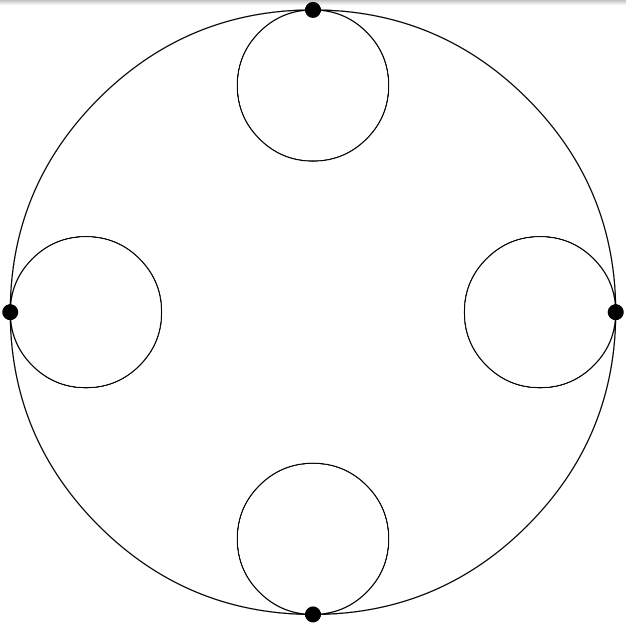

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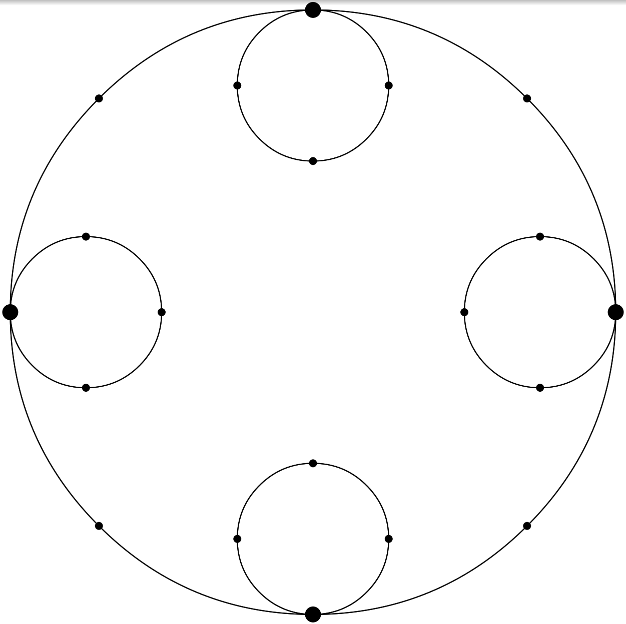

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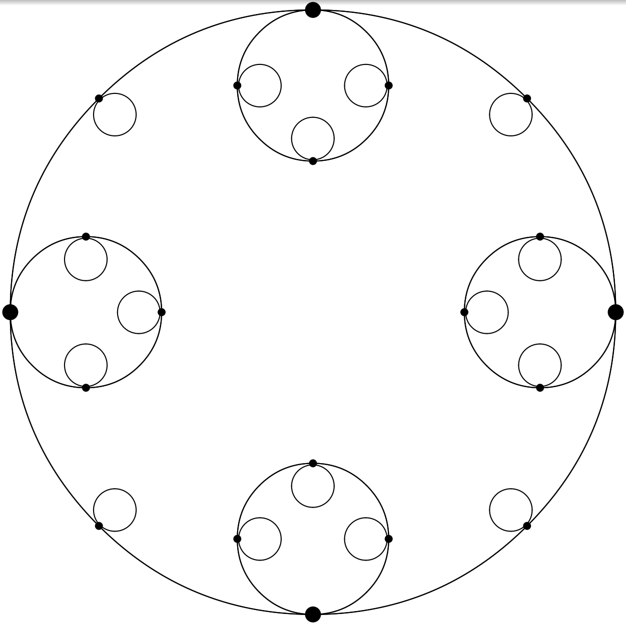
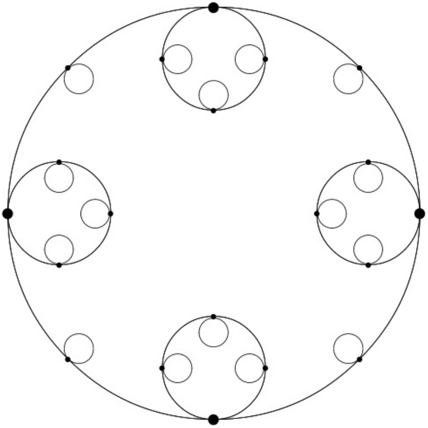


My action on μ^1



Dranishnikov's action