Bourgin-Yang Theorem for G-equivariant maps

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Joint work with
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INTRODUCTION

The classical Borsuk-Ulam theorem says that: $\forall$ continuous map f from the sphere $S(\mathbb{R}^{m+1})$ into $\mathbb{R}^m$,

$$\exists x \in S(\mathbb{R}^{m+1}) \text{ such that } f(x) = f(-x).$$

Remark. Let us observe that if $f : S(\mathbb{R}^m) \to \mathbb{R}^n$ is a continuous map and

$$A(f) = \{ x \in S(\mathbb{R}^m) \mid f(x) = f(-x) \},$$

we can define an equivariant map

$$\begin{aligned} h : \quad S(\mathbb{R}^m) &\to \mathbb{R}^n \\ x &\mapsto f(x) - f(-x) \end{aligned}$$

for which

$$Z_h = h^{-1}(0) = \{ x \in S(\mathbb{R}^m) \mid h(x) = f(x) - f(-x) = 0 \} = A(f).$$

Then, to give an estimate for a covering dimension of $A(f)$ it is enough to give an estimate for a covering dimension …

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Bourgin-Yang Theorem Let $f : S(\mathbb{R}^m) \to \mathbb{R}^n$ be a equivariant map. Then,

$$\dim Z_f \geq m - n - 1$$

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- To prove this theorem it is enough to show that $H^{m-n-1}(Z_f/\mathbb{Z}_2) \neq 0$, which implies $\dim Z_f \geq m - n - 1$.
- The situation for $\mathbb{Z}_p$, p prime, is analogous. [Munkholm, Izydorek & Rybicki, de Mattos & dos Santos, ...]

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We study the B-Y problem for G, as follows.

• $G = \mathbb{Z}_{p^k}$, p prime (equivariant K-theory).

Bourgin-Yang version of the Borsuk-Ulam theorem for $\mathbb{Z}_{p^k}$-equivariant maps, Algebraic and Geometric Topology 12 (2012) 2146 - 2151.

• $G = (\mathbb{Z}_p)^k$ be the p-torus of rank k, p a prime, or $G = \mathbb{T}^k = (S^1)^k$ be a k-dimensional torus (Borel cohomology, Borel localization theorem, Borel cohomology of stable cohomotopy theory).

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- Let V, W be two orthogonal representations of G such that $V^G = W^G = \{0\}$ (fixed points of G).

- If p is odd, V and W admit the complex structure. Put $d(V) = \dim_{\mathbb{C}} V = \frac{1}{2}\dim_{\mathbb{R}} V$ and the same for $d(W)$.

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Our main results is the following

Theorem. Let V, W be two orthogonal representations of $\mathbb{Z}_{p^k}$. Let $f : S(V) \to W$ be a G-equivariant map and $Z_f := f^{-1}(0)$. Then, $\dim(Z_f) = \dim(Z_f/G) \geq \phi(V, W)$, where ϕ is a function which we describe later. In particular, if

$$d(W) < d(V)p^{k-1}$$

then $\phi(V, W) \geq 0$ (e.g. it implies that there is no G-equivariant map from $S(V)$ into $S(W)$).

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Remark

Note that opposite to the case $G = \mathbb{Z}_p$, for $G = \mathbb{Z}_{p^k}$ the classical formulation of the Borsuk-Ulam theorem does not hold.

In Bartsch Lecture Notes [3] (Th. 3.22) it is shown that for any (finite) group G which contains an element g of order p^2 there exist two complex representations V, W, $\dim V > \dim W$, and an equivariant map $f : S(V) \to S(W)$.

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Anyway in [2] Bartsch has shown that there is not a G-equivariant map from $S(V)$ into $S(W)$ if

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He used an kind of cup-length theory defined in the equivariant K-theory, we shall use to.

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Let $\mathcal{A}$ be a set of G-spaces (usually, it is a family of orbits).
Let X be a G-spaces.

Definition

The $(\mathcal{A}, K_G^*)$ – cup length of X is the **smallest** r such that there exist $A_1, A_2, \ldots, A_r \in \mathcal{A}$ and G-maps

$$\beta_i : A_i \to X, \ 1 \leq i \leq r,$$

with the following property:

$\forall \ \gamma \in K_G^*(X) \ and \ \forall \ \omega_i \in \ker \beta_i^*,$

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- We can consider the notion of $(\mathcal{A}, K_G^*, I)$ – *length index* defined in a little bit different way.
- For the equivariant cohomology theory K_G^* and a G-space X, the cohomology
$$K_G^*(X)$$
is a R-module over the coefficient ring $R =: K_G^*(\mathrm{pt})$ via the natural G-map $p_X : X \to \mathrm{pt}$.
- We write
$$\omega \cdot \gamma = p_X^*(\omega) \cup \gamma, \text{ and } \omega_1 \cdot \omega_2 = \omega_1 \cup \omega_2,$$
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$$\omega_1 \cdot \omega_2 \cdot \cdots \cdot \omega_r \cdot \gamma = 0 \in K_G^*(X).$$

Taking the ideal $I := K_G(\mathrm{pt}) = R(G) \subset K_G^*(\mathrm{pt}) = R$

Definition.

The $(\mathcal{A}, K_G^*, I)$ – *length index* of X is the **smallest** r such that there exist $A_1, A_2, \ldots, A_r \in \mathcal{A}$ with the following property:

$\forall\ \gamma \in K_G^*(X)$ and $\forall\ \omega_i \in I \cap \ker(R \to K_G^*(A_i)) = \ker(K_G(\mathrm{pt}) \to K_G(A_i))$,
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- Given two powers $1 \leq m \leq n \leq p^{k-1}$ of p, we put

$$\mathcal{A}_{m,n} := \{G/H \mid H \subset G;\ m \leq |H| \leq n\}, \qquad (1)$$

where $|H|$ is the cardinality of H.

- Next we put

$$l_n(X) = (\mathcal{A}_{m,n}, K_G^*, I) - \textit{length index of}\ \ (X)\,. \qquad (2)$$

- The index l_n does not depend on m.

Taking $\mathcal{A}' = \{G/H \mid |H| = n\}$ we can show that

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- The following result of Bartsch ([2]) is fundamental for the **estimate from below of the index of Z_f**.
- We denote by $\mathcal{A}_X$ a set of all G-orbits of X.

Theorem A.

Let V be an orthogonal representation of $G = \mathbb{Z}_{p^k}$ with $V^G = \{0\}$ and $d = d(V) = \frac{1}{2}\dim_{\mathbb{R}} V$. Fix m, n two powers of p as above.

Then

$$i_n(S(V)) \geq \begin{cases} 1 + \left\lceil \frac{(d-1)m}{n} \right\rceil & \text{if } \mathcal{A}_{S(V)} \subset \mathcal{A}_{m,n} \\ \infty & \text{if } \mathcal{A}_{S(V)} \not\subset \mathcal{A}_{1,n} \end{cases}$$

where $\lceil x \rceil$ denotes the least integer greater than or equal to x. Moreover, if $\mathcal{A}_{S(V)} \subset \mathcal{A}_{n,n}$ then

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Relation between the *length index* and dimension

How to relate the length index in $K_G(X)$ and the covering dimension of X ?

G. Segal showed that there exists an Atiyah-Hirzebruch spectral sequence for the equivariant K-theory

$$E_2^{p,q} = H^p(X/G; \mathcal{K}_G^q \pi) \Rightarrow K_G^*(X),$$

where $\mathcal{K}_G^q$ is the sheaf on X/G associated to the presheaf $V \to K_G^q(\pi^{-1}\overline{V})$ ($\pi : X \to X/G$ is the projection) with the stalk $\mathcal{K}_G^q$ at an orbit $Gx = G/G_x$ equal to $R(G_x)$, if q is even, and $\mathcal{K}_G^q = 0$, if q is odd.

Consider the filtration of $K_G^*(X)$ associated to this spectral sequence:we

$$K_G^*(X) \supset K_{G,1}^*(X) \supset \ldots \supset K_{G,p}^*(X) \supset \ldots .$$

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(ii) $(K_{G,2}(X))^r = (K^0_{G,2}(X))^r = 0$.

Lemma B. *If the subgroups of G are totally ordered and H is the largest isotropic subgroup on X, then*

$$K_{G,1}(X) = ker(K_G(X) \to K_G(G/H)).$$

Theorem B. Let X be a compact G-space, with $G = \mathbb{Z}_{p^k}$, *and suppose that* $\mathcal{A}_X \subset \mathcal{A}_{m,n}$. *If* $l_n(X) \geq r + 1$ *then* $\dim X = \dim X/G \geq 2r$.

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Theorem 1.

Let V, W be two complex orthogonal representations of $G = \mathbb{Z}_{p^k}$, p prime, such that $V^G = W^G = \{0\}$. Consider $f : S(V) \xrightarrow{G} W$ an equivariant map. Suppose $\mathcal{A}_{S(V)} \subset \mathcal{A}_{m,n}$ and $\mathcal{A}_{S(W)} \subset \mathcal{A}_{m,n}$.

Then

$$l_n(Z_f) \geq 1 + \left[\frac{(d(V)-1)m}{n}\right] - d(W).$$

Consequently,

$$\dim(Z_f) \geq 2\Big(\left[\frac{(d(V)-1)m}{n}\right] - d(W)\Big) := \phi(V, W).$$

In particular, if $d(W) < d(V)/p^{k-1}$, then $\phi(V, W) \geq 0$, which means that there is no G-equivariant map from $S(V)$ into $S(W)$.

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Idea of the proof.

- Using monotonicity and additivity properties of the length index we show that

$$l_n(Z_f) \geq l_n(S(V)) - l_n(S(W)).$$

- By assumption, $\mathcal{A}_{S(V)} \subset \mathcal{A}_{m,n}$ and from Theorem A

$$l_n(Z_f) \geq 1 + \left[\frac{(d(V)-1)m}{n}\right] - d(W).$$

- Consequently, from Theorem B,

$$\dim(Z_f) \;\geq\; 2\Big(\left[\frac{(d(V)-1)m}{n}\right] - d(W)\Big) := \phi(V, W).$$

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$$l_n(Z_f) \geq l_n(S(V)) - l_n(S(W)).$$

- By assumption, $\mathcal{A}_{S(V)} \subset \mathcal{A}_{m,n}$ and from Theorem A

$$l_n(Z_f) \geq 1 + \left[\frac{(d(V)-1)m}{n}\right] - d(W).$$

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Theorem 2.

Let V, W be two real orthogonal representations of $G = \mathbb{Z}_{2^k}$, with $V^G = W^G = \{0\}$. Consider $f : S(V) \xrightarrow{G} W$ an equivariant map. Suppose that $\mathcal{A}_{S(V)} \subset \mathcal{A}_{m,n}$ and $\mathcal{A}_{S(W)} \subset \mathcal{A}_{m,n}$.

Then

$$\dim(Z_f) \geq \left[\frac{(d(V)-1)m}{n}\right] - d(W) = \phi(V, W).$$

In particular, if $d(W) < d(V)/2^{k-1}$, then $\phi(V, W) \geq 0$, which means that there is no G-equivariant map from $S(V)$ into $S(W)$.

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