Growth and Mahler measures in geometry and topology

Abstracts

1. Mikhail Belolipetsky

   Title: Lehmer's question, hyperbolic volumes and Margulis numbers.

   Abstract: I will discuss some connections between the three topics in the title of the talk. These connections are generally known but often overlooked. I will also consider some quantitative links between these topics which can be useful for the applications.

2. Michelle Bucher-Karlsson

   Title: Exponential growth rates of free and amalgamated products

   Abstract: I will show that there is a gap between square root of 2 and the golden ratio for the exponential growth rate of free products $G = A * B$ not isomorphic to the infinite dihedral group. For amalgamated products $G = A *_C B$ with $([A : C] - 1)([B : C] - 1) \geq 2$, I will show that lower exponential growth rate of the amalgamated product $\mathrm{PGL}(2, Z)$ is equal to the unique positive root of the polynomial $z^3 - z - 1$, which is smaller than square root of 2. This answers two questions by Avinoam Mann. Joint work with Alexey Talambutsa.

3. Abhijit Champanerkar

   Title: Mahler measure of the A-polynomial

   Abstract: Computing Mahler measure of two-variable polynomials is significantly harder than the one variable case. Boyd and Rodriguez-Villegas developed a technique for this which works for a family of polynomials called tempered polynomials. It turns out that A-polynomials naturally belong to this family. The Mahler measure, in this case, is a sum of dilogarithms and hence hyperbolic volumes. In this expository talk I will explain their technique from a topologists viewpoint and do examples.

4. Pierre Dehornoy

   Title: Bounds on the eigenvalues of the monodromy of a Lorenz knot.

   Abstract: Lorenz knots are special iterated plumbings of positive Hopf bands. The associated monodromy is then a product of positive Dehn twists. We will see how the particular plumbing scheme allows to compute the homological monodromy, and then to bound its growth rate.

5. Vincent Emery

   Title: Bounds for torsion homology of arithmetic groups.

Abstract: I will explain how results of Gelander can be used to obtain upper bounds for the torsion in homology of nonuniform arithmetic groups. These upper bounds are linear with respect to the covolume and are independent of the commensurability class. If time permits, I will discuss an application to the study of K-theory of number fields.

6. Yohei Komori

   Title: Arithmetic aspects of growth rates for hyperbolic Coxeter groups

   Abstract: Growth functions of Coxeter groups are computable by means of Steinberg formula and many people studied growth rates of them numerically and theoretically. In this talk we will review arithmetic properties of growth ratesfor 2 and 3 dimensional hyperbolic Coxeter groups, and report new results on 3 and 4 dimensional cases.

7. Thang Le

   Title: On the growth of torsions and regulators in finite coverings

   Abstarct: We discuss the growth of homology of finite coverings, with emphasis on (1) 3-manifolds and (2) abelian coverings of finite CW-complexex.

8. Chris Leininger Title: Dynamics on free-by-cyclic groups

   Abstract: For a free group F_n, the fully irreducible, hyperbolic automorphisms of F_n are a natural analogue of the pseudo-Anosov mapping classes of a surface. We describe a model for a free-by-cyclic group determined by a fully irreducible, hyperbolic automorphism that enjoys properties similar to those of the mapping torus of a pseudo-Anosov homeomorphism proved by Thurston and Fried. As a corollary, we construct fully irreducible, hyperbolic automorphisms of F_n with small stretch factor (i.e. on the order of $1/n$), analogous to the construction of McMullen for surfacess, complementing the work of Algom-Kfir and Rafi. This is joint work with S. Dowdall and I. Kapovich.

9. Wolfgang Lueck

   Title: Survey on Fulgede-Kadison determinants and L^2-torsion

   Abstract: The notion of a Fuglede-Kadison determinant is defined for group von Neumann algebras. In the case that the group is abelian, it is closely related to Mahler measures. The (generalized) Fuglede-Kadison determinant plays a role in the theory of von Neumann algebras and in the definition of L^2-torsion. We want to give a survey about these notions, explain the main known results, and discuss open conjectures about approximation of these L^2-invariants their classical analogues, which are determinants and Reidemeister or Ray-Singer torsion.

10. Werner Mueller

    Title: Growth of torsion in the cohomology of arithmetic groups.

    Abstract: I will consider certain families of locally symmetric spaces, defined by arithmetic groups. Examples are hyperbolic manifolds. For local systems defined by rational representations of the underlying semisimple group, the cohomology is defined over the integers. The goal is to study the growth of the torsion in the cohomology for appropriate sequences of representations.

11. Kathleen Petersen

    Title: Trace Fields and Hyperbolic 3-manifolds

Abstract: The trace field of a finite volume hyperbolic 3-manifold is a number field. I will survey some known results and questions about these number fields and their minimal polynomials, including a connection with Lehmers conjecture. Ill also present some examples.

12. Jean Raimbault

    Title: Growth of torsion homology for sequences of hyperbolic three–manifolds

    Abstract: It is conjectured by N. Bergeron and A. Venkatesh and T. Le that for a given (compact, arithmetic) hyperbolic–three manifold there is a sequence of finite covers such that (i) the injectivity radius tends to infinity; (ii) the torsion part of the first homology has a size that grows exponentially with the volume (with rate $1/(6\pi)$). This is wide open at present; I will describe generalizations of this conjecture, including some cases where there are proven results.

13. Mehmet Haluk Sengun

    Title: Torsion in the homology of Bianchi groups

    Abstract: Bianchi groups are groups of the form SL(2,R) where R is the ring of integers of an imaginary quadratic field. They form an important class of arithmetic Kleinian groups and moreover they hold a key role for the development of the Langlands programme for GL(2) beyond the totally real fields. In this talk, I will discuss the nature of the torsion in the homology of Bianchi groups. Time permitting, I will talk about the importance of these torsion classes for number theory.

14. Peter Shalen

    Title: Diameter and Homology of Hyperbolic 3-Manifolds

    Abstract: The Mostow rigidity theorem implies that the topological type of a closed, orientable hyperbolic n-manifold M determines M up to isometry. Hence any quantitative geometric invariant of a hyperbolic n-manifold is in principle a topological invariant. This raises the question of how these geometrically defined invariants compare with more classical topological invariants. For example, if we define the covering radius of M at a point p to be $\mathrm{cov}_p M = \max_{x\in M} \mathrm{dist}(p, x)$, a standard elementary argument shows that $\mathrm{rank}\, \pi_1(M) \leq B \exp(2 \min_{p\in M} \mathrm{cov}_p M)$ for some universal constant B. (The rank of $\pi_1(M)$ is defined to be its minimal number of generators.) This in particular gives a bound for $\mathrm{diam}\, M = \max_{p\in M} \mathrm{cov}_p M$ in terms of rank($\pi_1(M)$.

    The best version of the elementary argument that I know gives a value of about 1000 for B. Using fancy methods, I have shown that if one asks for a bound only on the dimension of $H_1(M; \mathbf{Z}_2)$ rather than on $\mathrm{rank}\, \pi_1(M)$, one can obtain a much better constant. This result is in the course of being written up, but I believe I can now show that

    $$\dim_{\mathbf{Z}_2} H_1(M; \mathbf{Z}_2) < B_0 \exp(2 \min_{p\in M} \mathrm{cov}_p M)$$

    where

    $$B_0 = \frac{67\pi}{4V_8} + \frac{39}{4} = 24.112\ldots.$$

    Here $V_8 = 3.66386\ldots$ denotes the volume of a regular ideal octahedron in $\mathbf{H}^3$.

    The proof, which I will sketch in my talk, relies on earlier work by Culler and myself, and work due to Agol, Storm and W. Thurston. The new ingredients are in large part topological.

15. Daniel Silver

    Title: Fibered knots, entropy and twisted Alexander polynomials

    Abstract: Fibered knots are in many ways the simplest knots. For example, integral polynomials with small Mahler measure often arise as Alexander polynomials of fibered knots. New theorems of D. Wise and I. Agol as well as earlier results of S. Friedl and S. Vidussi give new insights about fibered knots, topological entropy of representation shifts and twisted Alexander polynomials.

16. Chris Sinclair

    Title: Dobrowolski's Lower Bound

    Abstract: The best known lower bound for the Mahler measure of a polynomial as a function of degree comes from an idea of E. Dobrowolski. Dobrowolski's lemma states that the resultant of a degree N integer polynomial with the degree N polynomial formed by raising its roots to the pth power is divisible by p^N for prime p. I'll give an overview of how this produces Dobrowolski's lower bound using an argument by Cantor and Straus. I'll also present a minor extension of Dobrowolski's lemma due to McKinnon, Hare and myself. My aim will be to present the material in such a way that might suggest where further number theoretic information may (hypothetically) be useful to improve this lower bound and/or resolve Lehmer's problem.

17. Chris Smyth

    Title: Salem numbers: Constructions, conjectures and connections.

    Abstract: I survey our current state of knowledge about Salem numbers, and discuss the contexts in which they appear.

18. Susan Williams

    Title: Lehmer's question from the perspectives of knot theory and dynamical systems

    Abstract: Lehmer's question can be formulated in terms of knots and Alexander polynomials. It is equivalent also to a question about pseudo-Anosov homeomorphisms. In this expository talk we discuss both the topological and dynamical systems approaches.