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Noise source distribution in supersonic jets

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Abstract

In a previous experiment, Schlinker, Laufer and Kaplan reported, for the first time, the observation of two distinct sources of supersonic jet mixing noise. It is believed that one of the observed noise sources is the large turbulence structures of the jet flow and the other observed noise source is the fine scale turbulence. Recently, Tam and Auriault developed a theory for the prediction of the radiated noise from the fine scale turbulence of jets. The purpose of this paper is to show that the theoretical predictions agree well with the experimental measurements of Schlinker et al. not only for the radiated noise, but also for the noise source distribution within the jet.

1. Introduction

During the early 1970s, a comprehensive experimental study of supersonic jet noise was conducted at the University of Southern California (USC) [1]. Part of the measured data was reported by Laufer et al. [2]. What sets this study apart from other investigations is that its objective was to measure not just the noise intensity and spectra but the sources of jet mixing noise. Measuring noise sources is difficult. It requires special equipment and special techniques. It is not usually done even today. Needless to say, knowing the sources of jet noise and their locations is important. It provides useful information towards possible development of noise suppression schemes or devices.

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In order to measure the noise source strength distribution in a jet, a directional microphone was developed at USC. This system consisted of a spherical reflector capable of focusing on a local region of the jet and measuring the radiated noise. The noise spectra and source distributions of three perfectly expanded supersonic jets at Mach number 1.47, 1.97 and 2.47 and at room temperature were measured. It was found that the noise source strength distributions of these jets radiating to the maximum noise direction (32.5°, 37.5° and 45.0°, respectively, from the exhaust direction) and to the sideline at 90° were distinctly different (see Fig. 1). Real time pressure signal measured by an omnidirectional microphone indicated that the sound radiated to these two directions were also distinctly different. The pressure signal at 90° resembled that of subsonic jets, being random but smooth. On the other hand, for the two higher Mach number jets the pressure signals in the maximum noise direction had shock-like spikes with steep rise time. These distinctive difference together with the difference in source locations led Laufer et al. [2] to conclude that there were two intrinsically different noise sources. The noise radiated by the two sources had distinctively different directional, spectral and real time characteristics.

There is now fairly general agreement that the large turbulence structures and the fine scale turbulence of a high speed jet flow are two distinct sources of jet noise. Tam et al. [4] found, after examining a very large set of jet noise data, that the far-field jet noise spectra, regardless of jet Mach number and jet temperature, invariably fit two similarity spectra. One spectrum fits all noise data radiated in the downstream direction close to the jet axis and the other fits all spectrum data in the sideline and upstream direction. That the two similarity spectra fit measured far-field noise spectra well is confirmed in the works of Refs. [5–9]. The downstream spectrum has characteristics that can be associated with noise radiated by large turbulence structures while the sideline spectrum has characteristics that can be associated with noise from fine scale turbulence.

Recently, Tam and Auriault [3] developed a semi-empirical theory for predicting the fine scale turbulence noise from high speed jets. The theory used turbulence information provided by the

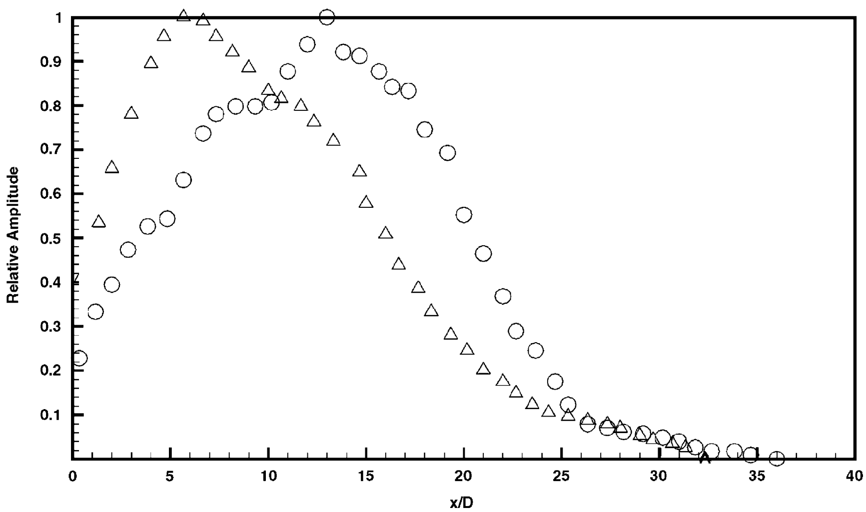

Fig. 1. Acoustic source strength per unit length along the jet axis measured by Schlinker [1], $M_j = 2.47$, $D = 1.0$ in, ○ 90°, △ 45°.

k–ε turbulence model. It contained three empirical constants. These constants were determined by best fit to a subset of jet noise data. It turns out, the Tam and Auriault theory can be used for computing the fine scale turbulence noise source distribution as well. This was not realized in the original development of the theory. The main purpose of this work is to compare the fine scale turbulence noise source distribution calculated by the Tam and Auriault theory with the experimental measurements of Schlinker et al. [1,2]. Some previously unpublished data will be included in the comparison. In addition, the predicted far-field noise spectra are used to compare with the noise measurements of Schlinker et al. at identical Mach numbers as the noise source distribution measurements. It is to be emphasized that the original values of the empirical constants of the Tam and Auriault theory are used in all the comparisons. In other words, the comparisons are made without any adjustment of the empirical constants that were deduced from lower Mach number jet noise data. It will be shown that good agreements are found both in the far-field noise spectra and in noise source distribution along the length of the jet. It is believed that the favorable agreements offer further support of the validity of the theory.

2. Comparison of measured noise data with similarity spectra

One objective of this work is to show that the noise from the two noise sources found in the experiments of Schlinker et al. [1,2] are the noise from the large turbulence structures and fine scale turbulence as proposed by Tam and coworkers [4,10,11]. To support this proposition, we will first demonstrate that the noise spectra measured by Schlinker in the maximum noise direction have the same shape as the similarity spectrum of the large turbulence structures noise, and that the spectra measured at 90° have the same shape as the similarity spectrum of the fine scale turbulence noise. Fig. 2 shows comparisons of the similarity spectrum of the large turbulence spectrum noise and the measured data in the maximum noise direction at Mach 1.47, 1.97 and

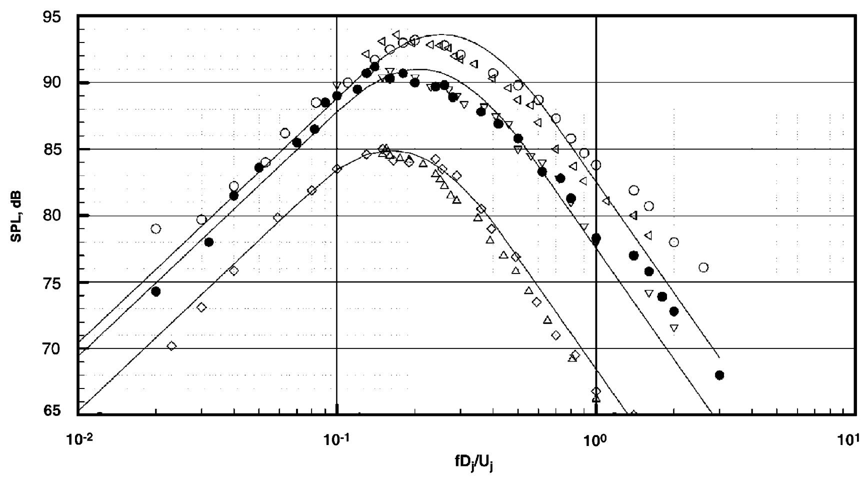

Fig. 2. Comparison between the shape of the large turbulence structures similarity noise spectrum and the experimental measurements of Schlinker [1] in the maximum noise direction. Mach 2.47, 1.97 and 1.47 cold jets. ○, ●, ◇ omnidirectional microphone; ◁, ▽, △ directional microphone. —— Similarity spectrum.

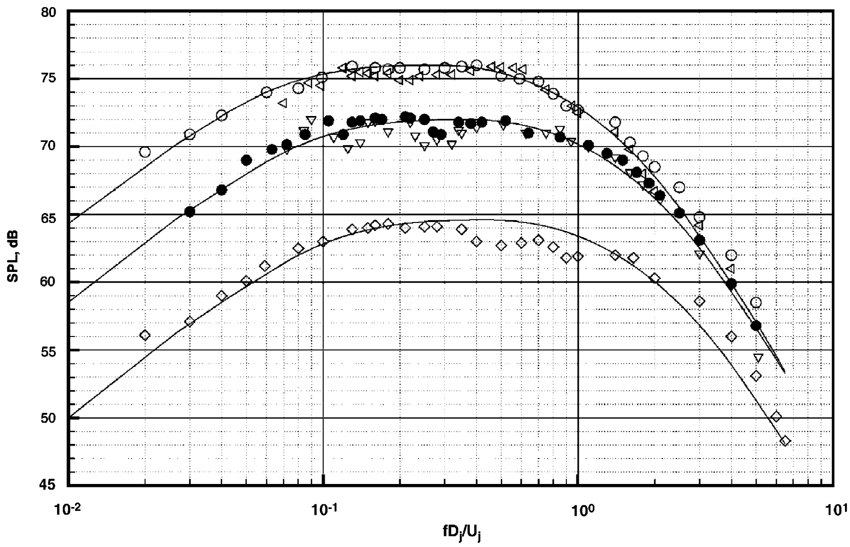

Fig. 3. Comparison between the shape of the fine scale turbulence similarity noise spectrum and the experimental measurements of Schlinker [1] at exhaust angle 90°. Mach 2.47, 1.97 and 1.47 cold jets. ○, ●, ◇, omnidirectional microphone; ◁, ▽ directional microphone. —— Similarity spectrum.

2.47. Fig. 3 shows comparisons of the similarity spectrum of the fine scale turbulence noise and measurements at the same Mach numbers in the 90° direction. As is evident, there is good agreement in every case. It is worthwhile to point out that the highest Mach number measurement that has been used previously to compare with the similarity spectrum was 2.0.

3. Comparisons between experimental measurements and fine scale turbulence noise theory

The main objective of this work is to compare the measurements of Schlinker to the Tam and Auriault fine scale turbulence noise theory [3]. In the theory, the source of noise is attributed to be the time rate of change of the turbulence kinetic energy. They reasoned that in gas kinetic theory, pressure is generated by the random velocity fluctuations of the molecules. It is equal to $\frac{2}{3}$ of the kinetic energy of the random motion. They made an analogy between random molecular motion and random motion of the fine scale turbulence. This led them to the conclusion that the fine scale turbulence motion would result in its exerting an effective turbulence pressure, equal to $\frac{2}{3}$ of the turbulence kinetic energy, on the surrounding fluid. Thus a change in the turbulence kinetic energy will cause pressure fluctuations in the jet flow. When the pressure fluctuations propagate to the far field, they becomes noise.

The Tam and Auriault theory provides a way to calculate the noise spectrum in absolute level. They demonstrated that the predictions of the theory were in good agreement with experimental measurements by Seiner in Ref. [4], Norum and Brown [12], Tanna et al. [13] and Ahuja [14] for cold and moderate temperature jets over the Mach number range of 0.3–2.0. Tam et al. [15] applied the theory to jets in simulated forward flight. They obtained good agreement with experimental measurements by Norum and Brown [12] and Plumblee [16] for both supersonic and subsonic jets at forward flight Mach number as large as 0.4. Subsequently, Tam and Pastouchenko [17] extended the application of the theory to nonaxisymmetric jets. They found good agreement with experimental data of Seiner et al. [18] and Tam and Zaman [9] for subsonic and supersonic jets issued from elliptic and rectangular nozzles.

In this section, we will first compare the 90° noise spectra measured by Schlinker [1] and the calculations by the Tam and Auriault theory. Good agreement will provide an indication that the sideline noise is generated by fine scale turbulence of the jet flow. We will then show that the Tam and Auriault theory can be used to calculate the noise source distribution inside the jet. We will compare these theoretical results with the extensive measurements of Schlinker. Both noise source distributions at selected frequencies and total noise source strength distributions are compared.

According to the Tam and Auriault theory, $S(R,\Theta,\phi,f)$, the fine scale turbulence noise spectrum (per unit frequency) measured at an observation point with spherical polar coordinates (R,Θ,ϕ), (the origin of the coordinate system is centered at the nozzle exit), is given by

$$S(R,\Theta,\phi,f) = (4\pi)^2\left(\frac{\pi}{\ln 2}\right)^{3/2}\int\int_{\text{jet}}\int \frac{\hat{q}_s^2\ell_s^3}{c^2\tau_s}\cdot\left\{\frac{\left|p_a(\mathbf{x}_2,\mathbf{x},\omega)\right|^2\exp\left[-\omega^2\ell_s^2/\bar{u}^2(4\ln 2)\right]}{1+\omega^2\tau_s^2\left(1-\left(\bar{u}/a_\infty\right)\cos\Theta\right)^2}\right\}\mathrm{d}\mathbf{x}_2, \quad (1)$$

where $\mathbf{x}_2$ is the source point, $\mathbf{x}$ is the far-field measurement point with spherical coordinates (R,Θ,ϕ), $p_a(\mathbf{x}_2,\mathbf{x},\omega)$ is the adjoint Green's function, $\omega=2\pi f$ is the angular frequency, $\bar{u}$ is the mean flow velocity at $\mathbf{x}_2$, a_∞ is the ambient sound speed, $\hat{q}_s$ is the fine scale turbulence intensity, ℓ_s is the eddy size, τ_s is the turbulence decay time. $\hat{q}_s$, ℓ_s and τ_s are related to k, the turbulence kinetic energy, and ε, the dissipation rate, of the k–ε models as follows:

$$\ell_s = c_\ell\ell = c_\ell\left(\frac{k^{3/2}}{\varepsilon}\right), \quad \tau_s = c_\tau\tau = c_\tau\left(\frac{k}{\varepsilon}\right)$$
$$\frac{\hat{q}_s}{c^2} = A^2q^2, \quad q = \frac{2}{3}\bar{\rho}k,$$

where $\bar{\rho}$ is the density of the mean flow at $\mathbf{x}_2$. c_ℓ, c_τ and A are the three empirical constants of the theory. Their numerical values as determined in Ref. [3] by best fit to the data are:

$$c_\ell = 0.256, \quad c_\tau = 0.233, \quad A = 0.755\,.$$

Fig. 4 shows a comparison of the noise spectra at 90° measured by Schlinker for jets at Mach 1.47, 1.97 and 2.47 and the calculated noise spectra using the Tam and Auriault theory. Unlike the comparisons shown in Fig. 3, there is no adjustment of constants in the calculation. The comparison is on an absolute level. As can be seen, there is good agreement in each case. The difference between prediction and measurements is within anticipated experimental uncertainty. We would like to point out that the highest Mach number experimental data that has been used in the past to compare with the Tam and Auriault theory is 2.0. The comparison at Mach 2.47 is new. This extends the known range of validity of the theory.

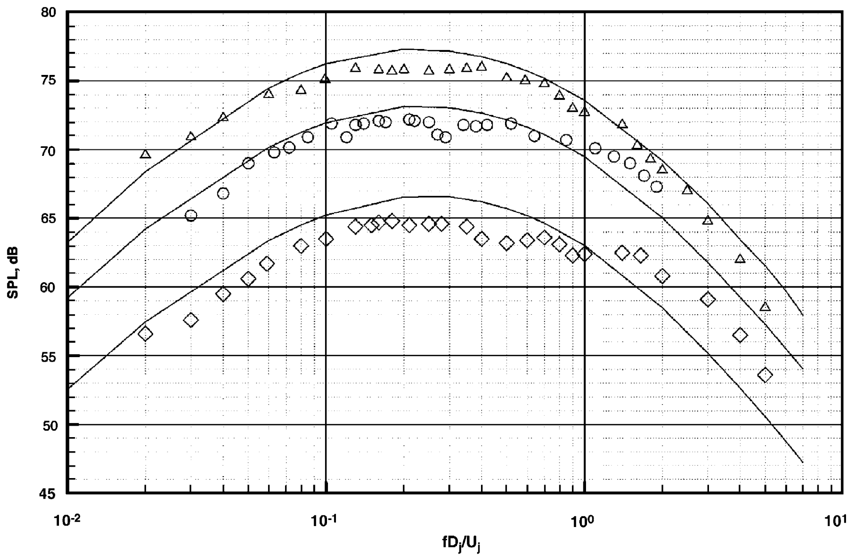

Fig. 4. Comparison between measured noise spectra [1] and theoretical predictions [3] at 90°. $D = 1.0$ in, $R = 81.5$ in. Jet Mach number 2.47, △; 1.97, ○; 1.47, ◇.

Let us denote the noise source strength per unit length of the jet for radiation to direction Θ at frequency f at x_2 by $S_x(R, \Theta, f, x_2)$. Then $S_x(R, \Theta, f, x_2)$ is related to $S(R, \Theta, \phi, f)$ by

$$S(R, \Theta, \phi, f) = \int_{\text{length of jet}} S_x(R, \Theta, f, x_2)\, \mathrm{d}x_2, \tag{2}$$

where $\mathbf{x}_2 = (r_2, \phi_2, x_2)$ in cylindrical coordinates. By comparing Eqs. (1) and (2), it is easy to find,

$$S_x(R, \Theta, f, x_2)$$
$$= (4\pi)^2 \left(\frac{\pi}{\ln 2}\right)^{3/2} \iint_{\text{cross-section of jet}} \frac{\hat{q}_s^2 \ell_s^3}{c^2 \tau_s} \left\{ \frac{\left|p_a(\mathbf{x}_2, \mathbf{x}, \omega)\right|^2 \exp\left[-\omega^2 \ell_s^2 / \bar{u}^2 (4 \ln 2)\right]}{1 + \omega^2 \tau_s^2 \left(1 - \left(\bar{u}/a_\infty\right) \cos \Theta\right)^2} \right\} r_2\, \mathrm{d}r_2\, \mathrm{d}\phi_2. \tag{3}$$

The total noise source strength per unit length of jet, $S_x(R, \Theta, x_2)$, is obtained by integrating $S_x(R, \Theta, f, x_2)$, over frequency. Thus

$$S_x(R, \Theta, x_2) = \int_0^\infty S_x(R, \Theta, f, x_2)\, \mathrm{d}f. \tag{4}$$

We will now compare the calculated distribution of total noise source strength (Eq. (4)) with the measurements of Schlinker [1]. Schlinker's data are given in relative source strength. Therefore, in what follows, the computed results are normalized to unity at the maximum location. Fig. 5 shows the measured and calculated relative total noise source strength distribution of the Mach 2.47 jet for radiation in the 90° direction ($\Theta = 90°$). In making this comparison and in all subsequent comparisons of noise source distribution, the measured data is shifted upstream by 4 diameters. This is done because of possible uncertainty in the experimental measurements and also in the jet mean flow calculations. Since the observation point is at a distance of 81 diameters from the jet,

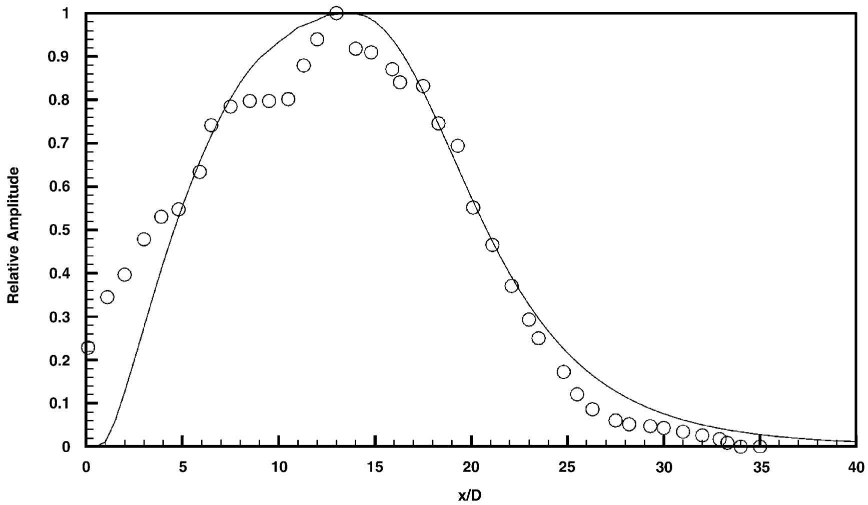

Fig. 5. Comparison between measured noise source strength distribution [1] and theoretical prediction, Eq. (4), for radiation at 90°. Mach 2.47 cold jet.

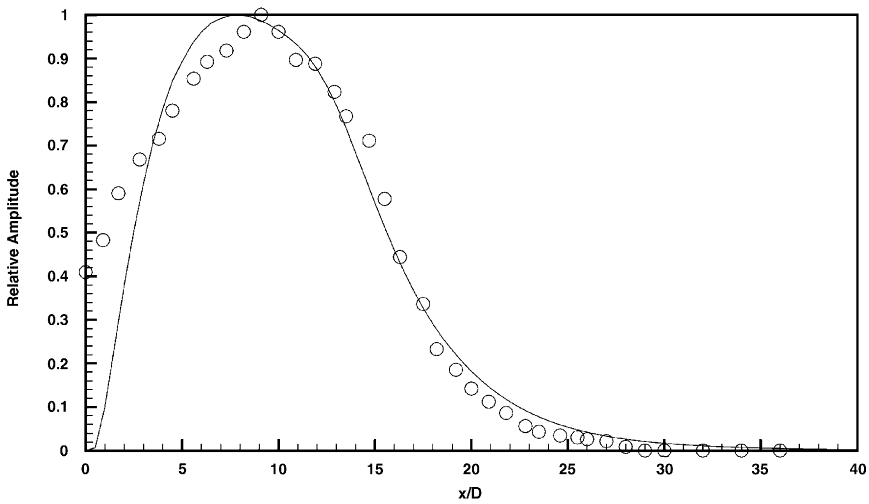

Fig. 6. Comparison between measured noise source strength distribution [1] and theoretical prediction, Eq. (4), for radiation at 90°. Mach 1.97 cold jet.

this shift amounts to a 2.8° bias in the alignment of the jet centerline and/or the zero angle reading of the spherical reflector. This is within the accuracy of the experiment. On the other hand, in computing the jet mean flow using the k–ε turbulence model, it is a practice to set the values of k and ε at the nozzle exit where the computation begins to very small values (the initial values of k and ε are not measured in most experiments). The result is that the computed mean flow in the mixing layer of the jet does not reach an equilibrium turbulent flow profile until a half to one diameter downstream. In principle, the location at which the equilibrium turbulent velocity profile

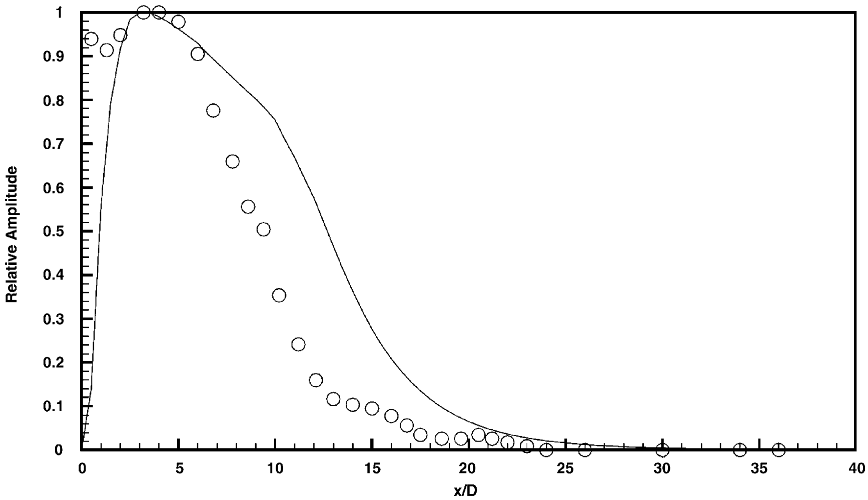

Fig. 7. Comparison between measured noise source strength distribution [1] and theoretical prediction, Eq. (4), for radiation at 90°. Mach 1.47 cold jet.

first appears should be considered as the origin of the downstream coordinate of the turbulent jet. As can be seen, there is good agreement between theory and experiment. Figs. 6 and 7 show similar comparisons for the Mach 1.97 and 1.47 jets. Again the locations of the peak noise source strength of the theory match those measured by Schlinker. There is also reasonable agreement in the overall shape of the distributions. Experimental measurements indicate a strong dependence of the location of the peak noise source on jet Mach number. At Mach 1.47, the peak is located at about 4 diameters downstream of the nozzle exit. At Mach 2.47 it is 13 diameters downstream. There is a shift of 9 diameters. That this is reproduced by the theory strongly suggests that the noise source is, indeed, the fine scale turbulence of the jet flow.

Fig. 8 shows the measured distribution of the relative amplitude of the noise source of the Mach 1.97 jet for noise radiated in the 90° direction at Strouhal number (fD/u_j) 0.09 (where D and u_j are the jet exit diameter and velocity, respectively). The calculated noise source distribution (Eq. (3)) is in good agreement with the measurement except beyond 25 diameters downstream. Fig. 9 shows a similar comparison of the noise source distribution at Strouhal number 0.4. The peak location is correctly calculated but the computed half-width of the distribution curve is about 25% larger. On considering both Figs. 8 and 9, it is easy to see that the higher frequency noise source is located closer to the nozzle exit as expected. The change in peak location with frequency is correctly calculated by the theory.

4. Summary

Schlinker and his coworkers [1,2] were the first to observe experimentally that there were two independent sources of supersonic jet mixing noise. In this work, we have demonstrated that the characteristics of their two noise sources are consistent with those of the two noise sources

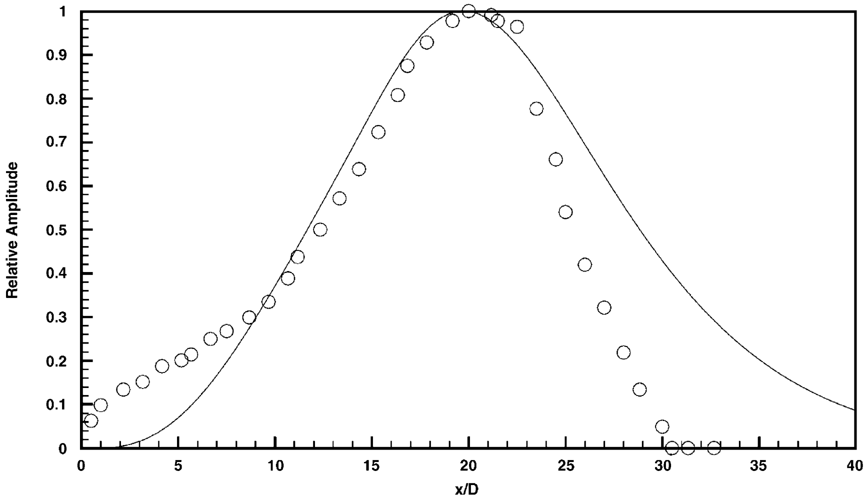

Fig. 8. Comparison between measured noise source distribution [1] at $St = 0.09$ and theoretical prediction, Eq. (3), for radiation at 90°. Mach 1.97 cold jet.

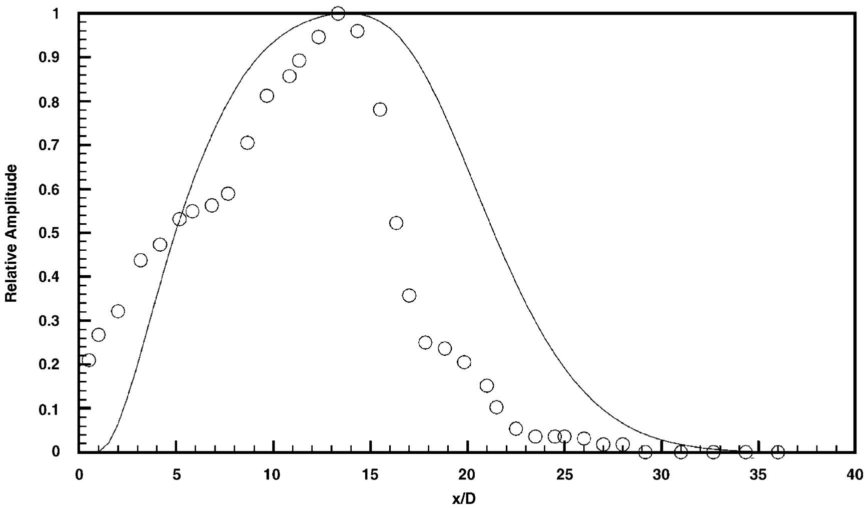

Fig. 9. Comparison between measured noise source distribution [1] at $St = 0.4$ and theoretical prediction, Eq. (3), for radiation at 90°. Mach 1.97 cold jet.

proposed by Tam and coworkers [4,10,11]. One of the noise sources is the large turbulence structures of the jet flow. Large turbulence structures radiate sound in the form of Mach waves. Mach waves are radiated in the downstream direction and are confined within the Mach cone. The other source of noise is the fine scale turbulence of the jet. This noise component has a weaker directivity pattern than that of the large turbulence structures. It is the dominant source of noise in all directions outside the Mach cone. We have also shown that the measured noise spectra inside the Mach cone at jet Mach number 2.47, 1.97 and 1.47 have the same shape as the similarity spectrum of the large turbulence structure noise found by Tam et al. [4]. At 90° the measured noise spectra are found to fit the similarity spectrum of the fine scale turbulence noise.

Data on noise source distributions in jets are not readily available in the literature. They are difficult and seldom measured. The data provided in Ref. [1] must, therefore, be regarded not only as important but also quite unique. We have shown for radiation at 90° direction, the measured data are in good agreement with the noise source strength distributions computed by the Tam and Auriault fine scale turbulence theory [3]. It is believed that this is the first extensive comparisons of noise source distributions between theory and experimental measurements. The good agreement provides strong support for the validity of the theory.

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