1. Let $A, B, C$ be sets. If $A \subseteq B \cup C$ then show $A - B \subseteq C$.

2. Let $p, q$ be statements. Which of the following statements are logically equivalent, if any? Which are tautologies, if any?

   - $S_1: p \lor q$
   - $S_2: (\neg q) \lor (p \implies q)$
   - $S_3: (\neg q) \implies p$

3. Give the definitions of:

   - (a) A function $f : A \to B$ is one-to-one (injective) when:
   
   - (b) The function $f^{-1} : P(B) \to P(A)$ is defined as follows: If $T \in P(B)$ then $x \in f^{-1}(T)$ if and only if:
   
   - (c) If $S \subseteq L$ where $L$ is a p.o.set with ordering $\leq$ then $u$ is a greatest lower bound for $S$ when:
   
   - (d) If $u$ is a bottom element of $S$, must $u$ then also be a greatest lower bound for $S$? (Yes/No with brief explanation).

4. Let $f : A \to B$ and $g : B \to C$. If the composition $g \circ f : A \to C$ is onto then show that $g$ is onto.

5. Suppose $L$ is a chain and that $S \subseteq L$ has no top element. Then show $\forall a \in S \exists b \in S \; b > a$. 

Test 1, Feb 6 2019, Intro Advanced Math.
Writing proofs.

1. **Direct proof for** \( p \implies q \).
   
   Assume: \( p \). To prove: \( q \).

2. **Proving** \( p \implies q \) **by contrapositive**.
   
   Assume: \( \neg q \). To prove: \( \neg p \).

3. **Proving** \( S \) **by contradiction**.
   
   Assume: \( \neg S \). To prove: a contradiction.

4. **Proving** \( p \implies q \) **by contradiction**.
   
   Assume: \( p \) and \( \neg q \). To prove: a contradiction.

5. **Direct proof for a** \( \forall x \in A P(x) \) **statement**.
   
   To ensure you prove \( P(x) \) for all (rather than for some) \( x \) in \( A \), do this:
   
   **Start your proof with:** Let \( x \in A \). To prove: \( P(x) \).

6. **Direct proof for** \( \exists x \in A P(x) \) **statement**.
   
   Take \( x := \) [write down an expression that is in \( A \), and satisfies \( P(x) \)].

7. **Proving** \( \forall x \in A P(x) \) **by contradiction**.
   
   Assume: \( x \in A \) and \( \neg P(x) \). To prove: a contradiction.

8. **Proving** \( \exists x \in A P(x) \) **by contradiction**.
   
   Assume: \( \neg P(x) \) for every \( x \in A \). To prove: a contradiction.

9. **Proving** \( S \) **by cases**.
   
   Suppose for example a statement \( p \) can help to prove \( S \). Write two proofs:
   
   Case 1: Assume \( p \). To prove: \( S \).
   
   Case 2: Assume \( \neg p \). To prove \( S \).

10. **Proving** \( p \land q \)
    
    Write two separate proofs: To prove: \( p \). To prove: \( q \).

11. **Proving** \( p \iff q \)
    
    Write two proofs. To prove: \( p \implies q \) To prove: \( q \implies p \).

12. **Proving** \( p \lor q \)
    
    Method (1): Assume \( \neg p \). To prove: \( q \).
    
    Method (2): Assume \( \neg q \). To prove: \( p \).
    
    Method (3): Assume \( \neg p \) and \( \neg q \). To prove: a contradiction.

13. **Using** \( p \lor q \) **to prove another statement** \( r \).
    
    Write two proofs:
    
    Assume \( p \). To prove \( r \).
    
    Assume \( q \). To prove \( r \).

14. **How to use a for-all statement** \( \forall x \in A P(x) \).
    
    You need to produce an element of \( A \), then use \( P \) for that element.