FindODE

find a linear ODE for a holonomic function

Calling Sequence

FindODE(f, v, maxorder)

Parameters

f	-	expression
v	-	either the dependent variable, of the form $y(x)$, or a *list* of two *names* $[Dx, x]$
maxorder	-	(optional) *posint*; maximal order of the ODE. Default value: 6

Description

- The input **f** should be a holonomic function, in other words, a function that satisfies a linear ODE with rational function coefficients. The **FindODE** function tries to find such an ODE.
- If **FindODE** fails to find a linear ODE, then $FAIL$ is returned.
- When using the $FindODE(f, y(x))$ calling sequence, the ODE will be returned in terms of the dependent variable $y(x)$, where x is the independent variable.
- When using the $FindODE(f, [Dx, x])$ calling sequence, the result will be given in differential operator form, that is, as a polynomial in the differential operator Dx whose coefficients are polynomials in the independent variable x.

See Also

DEtools
gfun[holexprtodiffeq]
LinearOperators
PDEtools[dpolyform]

- The second argument v can be omitted if the environment variable $\_Envdiffopdomain$ is assigned a list of two names, in which case the result will be given in differential operator notation.
- The resulting ODE will be cleared of denominators, that is, its coefficients are polynomials in the independent variable x without common factors.
- By default, **FindODE** incrementally searches for an ODE up to order 6. This maximal order can be overridden by specifying the optional third argument, **maxorder**.

Examples

> $with(DEtools):$

> $FindODE(\cos(\mathrm{sqrt}(x)), y(x))$

$$y(x) + 2\frac{d}{dx}y(x) + 4x\left(\frac{d^2}{dx^2}y(x)\right) \quad \textbf{(1)}$$

> $FindODE(\exp(x) + \mathrm{sqrt}(x), y(x))$

$$(2x+1)y(x) + \left(-4x^2 - 1\right)\left(\frac{d}{dx}y(x)\right) + \left(4x^2 - 2x\right)\left(\frac{d^2}{dx^2}y(x)\right) \quad \textbf{(2)}$$

The hypergeometric 2F1 equation.

> $FindODE(\mathrm{hypergeom}([a, b], [c], x), y(x))$

$$a\,b\,y(x) + (x\,a + x\,b - c + x)\left(\frac{d}{dx}y(x)\right) + \left(x^2 - x\right)\left(\frac{d^2}{dx^2}y(x)\right) \quad \textbf{(3)}$$

The tangent function is not holonomic, so the result is $FAIL$.

> $FindODE(\tan(x), y(x))$

$$FAIL \quad \textbf{(4)}$$

Operator notation.

> $FindODE(\cos(\mathrm{sqrt}(x)), [Dx, x])$

$$4\,x\,Dx^2 + 2\,Dx + 1 \quad \textbf{(5)}$$

The following example is a generating function from the Online Encyclopedia of Integer Sequences (http://oeis.org/A151357).

> $ogf := x^{-2}\left(Int\left(Int\left(\frac{2\,\mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right], [2], \frac{64\,x^3\,(1+x)}{(1-4\,x^2)^2}\right)}{(1-4\,x^2)^{\frac{3}{2}}}, x\right), x\right)\right)$

$$ogf := \frac{\int\int \frac{2\,\mathrm{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right], [2], \frac{64\,x^3\,(1+x)}{(-4\,x^2+1)^2}\right)}{(-4\,x^2+1)^{3/2}}\,\mathrm{d}x\,\mathrm{d}x}{x^2} \quad \textbf{(6)}$$

> $L := FindODE(ogf, [Dx, x])$

$$L := (192\,x^{10} + 640\,x^9 + 880\,x^8 + 656\,x^7 + 244\,x^6 + 16\,x^5 - 7\,x^4 - 3\,x^3)\,Dx^4 + (3072\,x^9 + 9792\,x^8 + 13344\,x^7 + 10016\,x^6 + 3632\,x^5 + 244\,x^4 - 86\,x^3 - 36\,x^2)\,Dx^3 + (13824\,x^8 + 42048\,x^7 + 56832\,x^6 + 42816\,x^5 + 15072\,x^4 + 1068\,x^3 - 264\,x^2 - 108\,x)\,Dx^2 + (18432\,x^7 + 53376\,x^6 + 71616\,x^5 + 53952\,x^4 + 18336\,x^3 + 1416\,x^2 - 180\,x - 72)\,Dx + 4608\,x^6 + 12672\,x^5 + 16896\,x^4 + 12672\,x^3 + 4128\,x^2 + 360\,x \quad \textbf{(7)}$$

> $DFactor(L, [Dx, x])$

$$\left[(192\,x^{10} + 640\,x^9 + 880\,x^8 + 656\,x^7 + 244\,x^6 + 16\,x^5 - 7\,x^4 - 3\,x^3)\left(Dx^2 + \left(2\,(1152\,x^7 + 3616\,x^6 + 4912\,x^5 + 3696\,x^4 + 1328\,x^3 + 90\,x^2 - 29\,x - 12)\,Dx\right) / \left(x\,(192\,x^7 + 640\,x^6 + 880\,x^5 + 656\,x^4 + 244\,x^3 + 16\,x^2 - 7\,x - 3)\right)\right.\right.$$

$$\left.\left. {}+ \ldots 40\,x^6 + 880\,x^5 + 656\,x^4 + 244\,x^3 + 16\,x^2 - 7\,x - 3))\right), Dx + \frac{2}{x}, Dx + \frac{2}{x}\right] \quad \textbf{(8)}$$

> $map(degree, \mathbf{(8)}, Dx)$

$$[2, 1, 1] \tag{9}$$

The first order factors of L come from the integrals. The second order factor comes from the hypergeometric 2F1 function.

Summing special functions preserves the order if they are contiguous (parameter differences are integers).

> $FindODE(\mathrm{BesselI}(0, x) + x\,\mathrm{BesselI}(2, x), y(x))$

$$\left(-x^4 - 2x^3 - 4x^2 - 3x\right) y(x) + \left(-x^3 + x + 4\right) \left(\frac{d}{dx} y(x)\right) + \left(x^4 + 2x^3 + x^2 + 2x\right) \left(\frac{d^2}{dx^2} y(x)\right) \tag{10}$$

Products of two contiguous second-order special functions satisfy third order equations.

> $FindODE(\mathrm{BesselI}(0, \mathrm{sqrt}(x))\,\mathrm{BesselI}(3, \mathrm{sqrt}(x)), [Dx, x])$

$$\left(24x^4 + 64x^3\right) Dx^3 + \left(84x^3 + 288x^2\right) Dx^2 + \left(-24x^3 - 130x^2 + 16x\right) Dx - 24x^2 - 179x - 216 \tag{11}$$

> $FindODE\left(\mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{2}{3}\right], [1], x\right)^2 - \mathrm{hypergeom}\left(\left[\frac{1}{3}, -\frac{1}{3}\right], [1], x\right) \mathrm{hypergeom}\left(\left[\frac{4}{3}, \frac{2}{3}\right], [1], x\right), [Dx, x]\right)$

$$\left(9x^5 - 27x^3 + 18x^2\right) Dx^3 + \left(45x^4 + 54x^3 - 135x^2 + 36x\right) Dx^2 + \left(38x^3 + 110x^2 - 94x\right) Dx + 2x^2 + 18x - 2 \tag{12}$$

Non-contiguous examples:

> $FindODE\left(\mathrm{KummerM}\left(\frac{1}{4}, 1, x\right) + \mathrm{KummerM}\left(-\frac{1}{4}, 1, x\right), [Dx, x]\right)$

$$16x^2 Dx^4 + \left(-32x^2 + 64x\right) Dx^3 + \left(16x^2 - 80x + 32\right) Dx^2 + (16x - 16) Dx - 1 \tag{13}$$

> $FindODE\left(\mathrm{LegendreP}\left(\frac{1}{4}, x\right) \mathrm{LegendreP}\left(\frac{1}{2}, x\right), y(x)\right)$

$$-495\, y(x) + 1440\, x \left(\frac{d}{dx} y(x)\right) + \left(5600x^2 - 1504\right) \left(\frac{d^2}{dx^2} y(x)\right) + \left(2560x^3 - 2560x\right) \left(\frac{d^3}{dx^3} y(x)\right) + \left(256x^4 - 512x^2\right. \tag{14}$$

$$+ 256) \left(\frac{d^4}{dx^4} y(x)\right)$$

> $dsolve(\mathbf{(14)})$

$$y(x) = \_C1 \, \mathrm{LegendreP}\left(\frac{1}{2}, x\right) \mathrm{LegendreQ}\left(\frac{1}{4}, x\right) + \_C2 \, \mathrm{LegendreP}\left(\frac{1}{4}, x\right) \mathrm{LegendreP}\left(\frac{1}{2}, x\right) + \_C3 \, \mathrm{LegendreQ}\left(\frac{1}{2}, x\right) \mathrm{LegendreP}\left(\frac{1}{4}, x\right) + \_C4 \, \mathrm{LegendreQ}\left(\frac{1}{2}, x\right) \mathrm{LegendreQ}\left(\frac{1}{4}, x\right) \quad \mathbf{(15)}$$

Compatibility

- The **DEtools[FindODE]** command was introduced in Maple 2019.
- For more information on Maple 2019 changes, see Updates in Maple 2019.