More sample questions.

1. Simplify the following cardinal numbers to the point where they look like $0, 1, 2, \ldots$, or $\aleph_0$, or c, or 2^c, or $2^{(2^c)}$, etc.

   $\mathrm{card}(\{3,3,3\}) =$
   $\mathrm{card}(P(\emptyset)) =$
   $\mathrm{card}(\mathbb{Q}) =$
   $\mathrm{card}(\mathbb{R} \times \mathbb{N}) =$
   $\aleph_0^{\aleph_0} =$
   $\mathrm{card}(P(\mathbb{R})) =$
   $c^{\aleph_0} =$

   Does there exist an injective function from $\mathbb{R}^{\mathbb{N}}$ to $\mathbb{R}$?

2. Give the definitions:

   (a) A function $f : A \to B$ is **injective** (same as "one-to-one") when:

   (b) Sets A and B have the **same cardinality** when:

3. If $f : A \to B$ and $g : B \to C$ are both **injective** (exercise 2a) then prove that the composition $g \circ f : A \to C$ is also **injective**.

4. Using **only** the definition from exercise 2b, show that $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ and $\mathbb{N}^* = \{1, 2, 3, \ldots\}$ have the **same cardinality**.

5. Let $A \subseteq B \subseteq C$ and suppose $\exists$ injective function from C to A. Then prove that there is a surjective function from B to C.

6. Let A, B be sets and let $C = A \times B$. Show that **at least one** of these must be true:
   (1) C is a **finite** set.
   or (2) There exists a **bijection** from A to C.
   or (3) There exists a **bijection** from B to C.