Handout OP: Organizing Proofs.

In math so far, you computed with numbers and functions. In this class you compute with statements. Organization is key to prevent computation errors.

In every statement $S$ that occurs in your proof, it must always be clear if $S$ is:

- Given
- To Prove (TP)
- Assumed
- Derived

Rules:

1. Never assume the TP statement (do not assume the conclusion).
2. Never prove Given or Assumed statements (the premises).
3. For any statement $S$ that occurs anywhere in your proof, it must be clearly indicated if it is Given, TP, Assumed, or Derived.
4. To indicate that $S$ is assumed, use the key word “Assume”.
5. Derived statements are all statements that you managed to prove from Given + Assumed statements. If $S$ is a derived statement, you must indicate that with key word(s). There are many key words you can choose from: hence, thus, therefore, so, because, it follows that, Then (as first word in a sentence, not as part of an if-then), etc.
6. Do not write $A \implies B$ to indicate that $B$ is derived from $A$.
   (A hence $B$ means both are true, but $A \implies B$ does not! See truth tables).
7. It is OK if an Assumed or Derived statement is false! If you correctly derived a false statement like $p \land \neg p$ then you proved that at least one of the assumptions is false. That can be useful, see WP#3,4,7,8.
8. Do not drop quantifiers! (regardless of whether they are written with symbols $\forall$ and $\exists$ or with key words like “for all” and “for some”). Suppose for example TP: $\forall x \in A \exists y \in B P(x,y)$.
   A direct proof starts with: “Let $x \in A$”.
   Errors on tests are often caused by a dropped quantifier. To minimize the chance of that, after “Let $x \in A$” write this: TP: $\exists y \in B P(x,y)$
   and then check which of WP#3 or WP#6 looks the most promising.
9. These three statements differ, do not mix them up:
   
   (1) $P(x)$ for all $x$ in $A$. (same as $\forall x \in A P(x)$) 
   (2) $P(x)$ for some $x$ in $A$. (same as $\exists x \in A P(x)$) 
   (3) $P(x)$ for no $x$ in $A$. (same as $\forall x \in A \neg P(x)$)
10. Do not use undeclared variables (to avoid mixups like in items 8 and 9).
11. To define $x$ (e.g. WP#6) don’t write $\ldots = x$. Instead write: Let $x := \ldots$
12. It helps a lot if you give each statement a separate line and label. If you use a given/assumed/derived statement that is not on the preceeding line, then cite its label. Example: Then $p$ by (1).
   That tells the reader that $p$ follows from the preceeding line and line (1).
   But it helps you too! (if you are stuck, look at given/assumed statements you have not yet used).
13. Use official formal definitions, not your interpretation of the definitions!