# Quiz 2, Intro Adv Math. Name:

1. Write down the definition of

   (a) The ***product*** of sets $A$ and $B$ is $A \times B =$

   (b) A relation $R$ is called ***transitive*** when:

   (c) Let $R$ b an equivalence relation on a set $A$. The ***equivalence class*** $E_x$ of an element $x \in A$ is:

2. Let $A, B, C$ be sets and assume that

$$C - A \subseteq B \tag{1}$$

Prove that then

$$C \subseteq A \bigcup B. \tag{2}$$

I will give part of the proof, then you will finish the proof. First of all, you ***must know*** that (2) is equivalent to (for all $x$):

$$x \in C \implies x \in A \bigcup B \tag{3}$$

Lets prove (3) with WP#4 which tells us to do this:
Assume $x \in C$ and $\neg(x \in A \bigcup B)$. $(*)$
To prove: a contradiction.

Now finish this proof. You may want to spell out line $(*)$ with De Morgan's law and then use another statement.
(only use assumed/given statements, do not use T.P. statements!)

Writing Proofs.

1. **Direct proof for $p \implies q$.**
   Assume: $p$. To prove: $q$.
2. **Proving $p \implies q$ by contrapositive**.
   Assume: $\neg q$. To prove: $\neg p$.
3. **Proving $S$ by contradiction**.
   Assume: $\neg S$. To prove: a contradiction.
4. **Proving $p \implies q$ by contradiction**.
   Assume: $p$ and $\neg q$. To prove: a contradiction.
5. **Direct proof for a $\forall_{x \in A} P(x)$ statement**.
   To ensure you prove $P(x)$ for *all* (rather than for *some*) $x$ in $A$, do this:
   **Start your proof with:** Let $x \in A$. To prove: $P(x)$.
6. **Direct proof for $\exists_{x \in A} P(x)$ statement**.
   Take $x :=$ [write down an expression that is in $A$, and satisfies $P(x)$].
7. **Proving $\forall_{x \in A} P(x)$ by contradiction.**
   Assume: $x \in A$ and $\neg P(x)$. To prove: a contradiction.
8. **Proving $\exists_{x \in A} P(x)$ by contradiction.**
   Assume: $\neg P(x)$ for every $x \in A$. To prove: a contradiction.
9. **Proving $S$ by cases**.
   Suppose for example a statement $p$ can help to prove $S$. Write two proofs:
   Case 1: Assume $p$. To prove: $S$.
   Case 2: Assume $\neg p$. To prove $S$.
10. **Proving $p \wedge q$**
    Write two separate proofs: To prove: $p$. To prove: $q$.
11. **Proving $p \iff q$**
    Write two proofs. To prove: $p \implies q$ To prove: $q \implies p$.
12. **Proving $p \vee q$**
    Method (1): Assume $\neg p$. To prove: $q$.
    Method (2): Assume $\neg q$. To prove: $p$.
    Method (3): Assume $\neg p$ and $\neg q$. To prove: a contradiction.
13. **Using $p \vee q$ to prove another statement $r$**.
    Write two proofs:
    Assume $p$. To prove $r$.
    Assume $q$. To prove $r$.
14. **How to use a for-all statement $\forall_{x \in A} P(x)$**.
    You need to produce an element of $A$, then use $P$ for that element.
15. If you want to **use an exists statement** like $\exists_{x \in A} P(x)$ to prove another statement, then you *may not choose* $x$. All you know is $x \in A$ and $P(x)$.