Quiz 2, Intro Adv Math. Name:

1. Write down the definition of
   (a) The **product** of sets $A$ and $B$ is $A \times B = \ldots$
   
   (b) A relation $R$ is called **transitive** when:

   (c) Let $R$ be an equivalence relation on a set $A$. The **equivalence class** $E_x$ of an element $x \in A$ is:

2. Let $A, B, C$ be sets and assume that
   
   \[ C - A \subseteq B \quad (1) \]
   
   Prove that then
   
   \[ C \subseteq A \cup B. \quad (2) \]
   
   I will give part of the proof, then you will finish the proof. First of all, you **must know** that (2) is equivalent to (for all $x$):
   
   \[ x \in C \implies x \in A \cup B \quad (3) \]
   
   Let’s prove (3) with WP#4 which tells us to do this:
   
   Assume $x \in C$ and $\neg (x \in A \cup B)$. \((*)\)
   
   To prove: a contradiction.
   
   Now finish this proof. You may want to spell out line \((*)\) with De Morgan’s law and then use another statement.
   
   (only use assumed/given statements, do not use T.P. statements!)
Writing Proofs.

1. **Direct proof for** \( p \implies q \).
   Assume: \( p \). To prove: \( q \).

2. **Proving** \( p \implies q \) **by contrapositive**.
   Assume: \( \neg q \). To prove: \( \neg p \).

3. **Proving** \( S \) **by contradiction**.
   Assume: \( \neg S \). To prove: a contradiction.

4. **Proving** \( p \implies q \) **by contradiction**.
   Assume: \( p \) and \( \neg q \). To prove: a contradiction.

5. **Direct proof for a** \( \forall x \in A P(x) \) **statement**.
   To ensure you prove \( P(x) \) for all (rather than for some) \( x \) in \( A \), do this:
   **Start your proof with:** Let \( x \in A \). To prove: \( P(x) \).

6. **Direct proof for** \( \exists x \in A P(x) \) **statement**.
   Take \( x := \) [write down an expression that is in \( A \), and satisfies \( P(x) \)].

7. **Proving** \( \forall x \in A P(x) \) **by contradiction**.
   Assume: \( x \in A \) and \( \neg P(x) \). To prove: a contradiction.

8. **Proving** \( \exists x \in A P(x) \) **by contradiction**.
   Assume: \( \neg P(x) \) for every \( x \in A \). To prove: a contradiction.

9. **Proving** \( S \) **by cases**.
   Suppose for example a statement \( p \) can help to prove \( S \). Write two proofs:
   Case 1: Assume \( p \). To prove: \( S \).
   Case 2: Assume \( \neg p \). To prove: \( S \).

10. **Proving** \( p \land q \)
    Write two separate proofs: To prove: \( p \). To prove: \( q \).

11. **Proving** \( p \iff q \)
    Write two proofs. To prove: \( p \implies q \) To prove: \( q \implies p \).

12. **Proving** \( p \lor q \)
    Method (1): Assume \( \neg p \). To prove: \( q \).
    Method (2): Assume \( \neg q \). To prove: \( p \).
    Method (3): Assume \( \neg p \) and \( \neg q \). To prove: a contradiction.

13. **Using** \( p \lor q \) **to prove another statement** \( r \).
    Write two proofs:
    Assume \( p \). To prove \( r \).
    Assume \( q \). To prove \( r \).

14. **How to use a for-all statement** \( \forall x \in A P(x) \).
    You need to produce an element of \( A \), then use \( P \) for that element.

15. If you want to use an **exists statement** like \( \exists x \in A P(x) \) to prove another statement, then you may not choose \( x \). All you know is \( x \in A \) and \( P(x) \).