Quiz 2, Intro Adv Math. Name:

1. Write down the definition of

   (a) The ***product*** of sets A and B is $A \times B =$

   $$\text{Answer}: \quad \{(a, b) \mid a \in A, b \in B\}$$

   Make sure you are familiar with set notation. The notation $\{x \mid P(x)\}$ (instead of $\mid$ you may also use a colon) means the set of all x for which $P(x)$ is true. Spelled out, the answer above is: the set of all pairs (a, b) for which $a \in A$ and $b \in B$.

   (b) A relation R is called ***transitive*** when:

   $$xRy \wedge yRz \Longrightarrow xRz$$

   (c) Let R b an equivalence relation on a set A. The ***equivalence class*** E_x of an element $x \in A$ is:

   $$\text{Answer}: \quad \{y \in A \mid xRy\}$$

   Again, make sure you can read and write set notation. If you spell out this answer in words you get: "the set of all elements of A that are related to x under the relation R". However:

   You are much better off if you ***do not spell things out like that***. It takes some effort getting used to short notation, but it will save time during HW and tests, and makes proofs easier to read + write.

2. Let A, B, C be sets and assume that

   $$C - A \subseteq B \tag{1}$$

   Prove that then

   $$C \subseteq A \bigcup B. \tag{2}$$

   I will give part of the proof, then you will finish the proof. First of all, you ***must know*** that (2) is equivalent to (for all x):

   $$x \in C \implies x \in A \bigcup B \tag{3}$$

   Lets prove (3) with WP#4 which tells us to do this:
   Assume $x \in C$ and $\neg(x \in A \bigcup B)$. $(*)$
   To prove: a contradiction.

   L1: (*) is equivalent to $x \in C$ and $x \notin A \wedge x \notin B$ (to see this, look at De Morgan's laws)
   So $x \in C$ and $x \notin A$, so $x \in C - A$ and then $x \in B$ by (1) but that contradicts the last item $x \notin B$ on L1.

Writing Proofs.

1. **Direct proof for $p \implies q$.**
   Assume: p. To prove: q.

2. **Proving $p \implies q$ by contrapositive**.
   Assume: $\neg q$. To prove: $\neg p$.

3. **Proving S by contradiction**.
   Assume: $\neg S$. To prove: a contradiction.

4. **Proving $p \implies q$ by contradiction**.
   Assume: p and $\neg q$. To prove: a contradiction.

5. **Direct proof for a $\forall_{x \in A} P(x)$ statement**.
   To ensure you prove $P(x)$ for *all* (rather than for *some*) x in A, do this:
   Start your proof with: Let $x \in A$. To prove: $P(x)$.

6. **Direct proof for $\exists_{x \in A} P(x)$ statement**.
   Take $x :=$ [write down an expression that is in A, and satisfies $P(x)$].

7. **Proving $\forall_{x \in A} P(x)$ by contradiction.**
   Assume: $x \in A$ and $\neg P(x)$. To prove: a contradiction.

8. **Proving $\exists_{x \in A} P(x)$ by contradiction.**
   Assume: $\neg P(x)$ for every $x \in A$. To prove: a contradiction.

9. **Proving S by cases**.
   Suppose for example a statement p can help to prove S. Write two proofs:
   Case 1: Assume p. To prove: S.
   Case 2: Assume $\neg p$. To prove S.

10. **Proving $p \wedge q$**
    Write two separate proofs: To prove: p. To prove: q.

11. **Proving $p \iff q$**
    Write two proofs. To prove: $p \implies q$ To prove: $q \implies p$.

12. **Proving $p \vee q$**
    Method (1): Assume $\neg p$. To prove: q.
    Method (2): Assume $\neg q$. To prove: p.
    Method (3): Assume $\neg p$ and $\neg q$. To prove: a contradiction.

13. **Using $p \vee q$ to prove another statement r**.
    Write two proofs:
    Assume p. To prove r.
    Assume q. To prove r.

14. **How to use a for-all statement $\forall_{x \in A} P(x)$**.
    You need to produce an element of A, then use P for that element.

15. If you want to **use an exists statement** like $\exists_{x \in A} P(x)$ to prove another statement, then you *may not choose* x. All you know is $x \in A$ and $P(x)$.