

Test 1 sample questions, Intro Advanced Math

1. Let p, q be statements. Which of the following statements are logically equivalent, if any? (list all, there may be more than one).

$$S_1 : p \implies q$$

$$S_2 : q \implies p$$

$$S_3 : p \vee \neg q$$

$$S_4 : (\neg q) \implies (\neg p)$$

$$S_5 : p \wedge \neg q.$$

2. Let p, q be statements. Determine the truth tables of the following two statements. Then check if these statements are logically equivalent or not.
Statement 1:

$$(p \vee q) \implies (\neg p \vee \neg q).$$

Statement 2:

$$(p \wedge q) \implies (\neg p \vee \neg q).$$

3. Let A, B, C be sets and assume that $C - B \subseteq C - A$. Prove that then $A \cap C \subseteq B$.
4. Let A, B, C be sets and assume that $C - A \subseteq B$.
Prove that then $C \subseteq A \cup B$.
5. Suppose that A is a set and that for every set B we have $A \subseteq B$.
Then prove that $A = \emptyset$.
Hint: Do these exercises with the Writing Proofs handout next to you.
WP#14 says that we should make a clever choice for B and then use the given statement ($A \subseteq B$) for that B .

6. Consider the statement S :

$$S : \quad \forall_{p \in \mathbb{R}} \quad p > 0 \implies 10^{11} \times p > 1$$

Write down $\neg S$, the negation of statement S .

For which of the following statements can you give a proof, S or $\neg S$?