Answers to sample questions

Read the handout "Writing proofs". Notation: WP#1 it means item 1 in that handout.

1. After building the truth-tables for $S_1, \ldots, S_5$ (let me know if you need help with that) I found that S_1 is logically equivalent with S_4, and that S_2 is logically equivalent with S_3.

2. If you need help writing truth-tables, let me know, then we'll do this one in class.

3. Let A, B, C be sets and assume that $C - B \subseteq C - A$. Prove $A \bigcap C \subseteq B$.

   Given is: $x \in C - B \Longrightarrow x \in C - A$. Lets call this statement (1).
   To prove: $x \in A \bigcap C \Longrightarrow x \in B$. Lets try a direct proof (WP#1):

   Let $x \in A \bigcap C$, so $x \in A$ and $x \in C$.
   To prove: $x \in B$.
   There are several ways to do this, lets try "Proof by contradiction" (WP#3):

   Assume: $x \notin B$.
   To prove: a contradiction.
   We assumed: $x \in A, x \in C, x \notin B$.
   So $x \in C - B$ but $x \notin C - A$, contradicting (1).

4. Let A, B, C be sets and assume that $C - A \subseteq B$.
   Prove that then $C \subseteq A \bigcup B$.

   Let $x \in C$.
   To prove: $x \in A \bigcup B$, in other words, x is in A or x is in B.

   There are several ways to do this, lets try proof by cases (WP#9).

   Distinguish two cases: Case 1: $x \in A$ and Case 2: $x \notin A$.
   In Case 1 we're already done.
   In Case 2, x is in $C - A$, and thus in B since $C - A \subseteq B$.

5. Make sure that when you work on proofs to have the WP (Writing Proofs) handout with you. If you lost this handout, you can print it from my website www.math.fsu.edu/∼hoeij (look under Fall 2019 and click on handouts).

   The hint was to use WP#14 which says if something is true for all cases (all B's in our exercise) then use the fact that it is true for a well-chosen case. So we should make a smart choice for B and then use the given information for that choice of B. We are told that $A \subseteq B$ for any set B. That includes the case $B = \emptyset$. So $A \subseteq \emptyset$. But by the exercise due today (ask if you couldn't do that one!) we saw that that implies $A = \emptyset$.

6. Lets look at this one in class.