Test 2 sample questions, Intro Advanced Math

1. Let R be a relation on a set S. Give the definition: R is a *partial ordering* when

   (a)

   (b)

   (c)

   Study note: The answer is on p.77 of the book. Where the book writes $(x, y) \in R$, it is OK to use the shorter notation xRy.

2. If R is a partial ordering, then R is a total ordering if it has one more property, namely:

   Study note: The answer is the item on p.78 that wasn't part of the previous answer.

3. Suppose that R is a partial ordering, but not a total ordering. Then show that there exists a non-empty subset $A \subseteq S$ for which A does not have a minimal element (in other words: R fails the condition written in the the last line in Definition 4.2.8 on page 78).

   Hints: The answer to the previous question was item (c) on page 78, which, written as a formula looks like $\forall_{x,y \in S}\ x \neq y \Longrightarrow xRy \vee yRx$.
   Now compute the negation of that formula. Is there a way to produce some subset of S from that?

4. Give the definition of an equivalence relation.

5. Let $f : A \to B$ be a function. We now define a relation R on A as follows: xRy is true if and only if $f(x) = f(y)$. Is this relation:

   (a) Reflexive?

   (b) Symmetric?

   (c) Transitive?

   (d) An equivalence relation?

   (e) If R is a partial ordering then prove that f is injective.

6. Suppose that $f : A \to B$ is not injective. Show that $\text{card}(A) \geq 2$.

7. Give a function $f : \mathbb{N}^* \to \mathbb{N}^*$ that is injective but not surjective.
   Give a function $g : \mathbb{N}^* \to \mathbb{N}^*$ that is surjective but not injective.

8. If A is any countably infinite set (item 5 in the handout) then show that there exists a function from A to A that is injective but not surjective.

9. If S is any infinite set, then use items 17 and 5 from the handout to show that there exists a function from S to S that is injective but not surjective.