Test 2 sample questions, Intro Advanced Math

1. Let $R$ be a relation on a set $S$. Give the definition: $R$ is a partial ordering when
   (a) 
   (b) 
   (c)

   Study note: The answer is on p.77 of the book. Where the book writes $(x, y) \in R$, it is OK to use the shorter notation $xRy$.

2. If $R$ is a partial ordering, then $R$ is a total ordering if it has one more property, namely:

   Study note: The answer is the item on p.78 that wasn’t part of the previous answer.

3. Suppose that $R$ is a partial ordering, but not a total ordering. Then show that there exists a non-empty subset $A \subseteq S$ for which $A$ does not have a minimal element (in other words: $R$ fails the condition written in the the last line in Definition 4.2.8 on page 78).

   Hints: The answer to the previous question was item (c) on page 78, which, written as a formula looks like $\forall x, y \in S \ x \neq y \implies xRy \lor yRx$.

   Now compute the negation of that formula. Is there a way to produce some subset of $S$ from that?

4. Give the definition of an equivalence relation.

5. Let $f : A \rightarrow B$ be a function. We now define a relation $R$ on $A$ as follows: $xRy$ is true if and only if $f(x) = f(y)$. Is this relation:
   (a) Reflexive?
   (b) Symmetric?
   (c) Transitive?
   (d) An equivalence relation?
   (e) If $R$ is a partial ordering then prove that $f$ is injective.

6. Suppose that $f : A \rightarrow B$ is not injective. Show that $\text{card}(A) \geq 2$.

7. Give a function $f : \mathbb{N}^* \rightarrow \mathbb{N}^*$ that is injective but not surjective.

   Give a function $g : \mathbb{N}^* \rightarrow \mathbb{N}^*$ that is surjective but not injective.

8. If $A$ is any countably infinite set (item 5 in the handout) then show that there exists a function from $A$ to $A$ that is injective but not surjective.

9. If $S$ is any infinite set, then use items 17 and 5 from the handout to show that there exists a function from $S$ to $S$ that is injective but not surjective.