GRV I, test 1, 9:05 - 9:55 am, Monday September 28, 2020.

1. Let σ be the following element of S_{10}.

$$\sigma = (1\ 2\ 3\ 4\ 5\ 6\ 7)(6\ 7\ 8\ 9\ 10).$$

   Compute a disjoint cycle notation for this permutation σ. Compute the order of σ.

2. Let G be a group with n elements, and suppose that $\gcd(k, n) = 1$. Prove that for every $g \in G$ there exists $h \in G$ for which $h^k = g$.
   (Hint: Bezout's equation)

3. Suppose that a, b, and ab each have order 2. Show that a and b commute.

4. Let G be a group and let H be a subgroup of index 2 (so G/H has 2 elements, H and $G - H$). Show that H is a normal subgroup of G.

5. Let G is a group. Let $A = G$ and let G act on A by conjugation, so $g \cdot a = gag^{-1}$. Now fix one element $a \in A$.
   Let $\mathcal{O} := \{g \cdot a \mid g \in G\} = \{gag^{-1} \mid g \in G\}$.

   (a) If $g_1 \cdot a = g_2 \cdot a$ then show that $g_1 \in g_2\, C_G(a)$.

   (b) Show $|\mathcal{O}| = |G : C_G(a)|$.